Part V

Conclusion

Chapter

14

A Broad Concept of Social Justice

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> *"Human well-being: A context- and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience."*
>
> —International Council of Scientific Unions (2010, p. 20)

In this concluding chapter, I encourage mathematics educators to think about a broader conception of social justice, one that aims toward human well-being, as stated in the International Council of Scientific Unions (ICSU) statement above. Doing so leads to a broader vision of teaching mathematics for social justice (TMfSJ).

I understand that this book is addressed primarily to mathematics teachers and teacher educators and that it is intended to be used in both preservice and in-service courses and professional development programs. As noted by the editors, the purpose of this book is: (a) to provide a historical perspective and theoretical grounding for critical mathematics in general and TMfSJ in particular, (b) to demonstrate how TMfSJ might be integrated into preservice and in-service mathematics teacher education and professional development, and (c) to demonstrate what TMfSJ might "look like" within mathematics classrooms. Before writing this closing chapter, I had the privilege of reading the previous chapters, which cover well the three purposes of the book. I am impressed by the broad coverage of the various chapters. Relying largely on personal experiences, the contributing authors clearly point to mathematics as a powerful instrument for achieving social justice. Similar to my colleagues, I feel that as mathematicians and mathematics educators we have a distinctive commitment to the major objective of educating for social justice. For nearly fifty years now, I have been discussing proposals of mathematics for social justice, with different formulations, as the main focus of mathematics education.

I believe that mathematics educators should be educators who regard mathematics as an important instrument to prepare future generations to live in a world with peace and human dignity for all. I see it as a mistaken view if one considers the main objective of mathematics educators to be transmitting mathematics without reference to the ethics of its uses. Although mathematics is taught with the declared intention that it will be useful for everyday life, mathematics educators cannot ignore the fact that their most successful students may be engineers who design lethal weapons or reinforce the practices of brutal capitalism. Without a clear understanding of how mathematics can help in attaining peace and human dignity for all, and thus social justice, mathematics educators may miss an important ethical responsibility.

In 1976, in my controversial discussion paper on why do we teach mathematics, presented at the Third International Congress of Mathematical Education (ICME-3), in Karlsruhe, Germany, I stated:

> We see the educational process as the conjugation of global socio-economic aspects aiming at the betterment of the quality of life. In this conjugation intervene, the same as in the technological process, the philosophy to which society subscribes, as well as considerations about manpower and available material resources. (D'Ambrosio 1976, p. 224)

Almost twenty years later, in 1993, at the fifteenth annual conference of the North American Chapter of the Psychology of Mathematics Education (PME-NA), in Pacific Grove, California, I went further in these ideas:

> Although the main concern of this meeting is Mathematics Education, I believe I will be allowed to subordinate my comments to a higher objective: the survival of civilization on Earth with dignity for all. This is not merely jargonizing. The world is threatened, not only by aggressions against nature and the environment. We are equally concerned with increasing violations of human dignity. We face more and more cases of life under fear, hatred and violation of the basic principles upon which civilization rests. (D'Ambrosio 1993, p. 31)

In other writings, I have asked for a new thinking in mathematics education (see, e.g., D'Ambrosio 1985, 1990, 1998, 2001a, 2001b). My objective, in this concluding chapter, is to stress the fact that our most urgent concern is to teach mathematics for social justice as it is understood in the broad sense of attaining human well-being, which comprises the basic components of a good life: freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience.

In the words of Koestler (this volume) and others, mathematics acts as a "gatekeeper" to economic success, to active citizenship, and to higher education in our society. Indeed, mathematics is present in all the major achievements of civilization. Advances in mathematics are associated with progress. But, paradoxically, mathematics has been the main instrument in weaponry and in economics. I have often referred to mathematics as the imprint of modern society, for good and evil. As historian Mary Lefkowitz says, "The evolution of general mathematical theories from those basics [the mathematics of Egyptians, Sumerians, and others] is *the real basis of Western thought*" (as quoted in Ringle 1996, emphasis added).

As a consequence of wars, of greedy capitalism, and of uncontrollable consumerism, people are killed in a broad sense, either physically or morally, as the termination of life and also as the loss of dignity. I understand the violation of social justice in this broad conception. As the political scientist Glenn Paige (2002) argues:

> There are no social relationships that require actual or threatened killing to sustain or change them. No relationships of dominance or exclusion—boundaries, forms of government, property, gender, race, ethnicity, class, or systems of spiritual or secular belief—require killing to support or challenge them. This does not assume that such a society is unbounded, undifferentiated, or conflict-free, but only that its structure and processes do not derive from or depend upon killing. There are no vocations, legitimate or illegitimate, whose purpose is to kill. (p. 30)

Similarly, the *Charter For a World Without Violence* (http://www.nobelforpeace-summits.org/charter-for-a-world-without-violence-2/), endorsed by Nobel laureates, ends with the appeal:

> To address all forms of violence we encourage scientific research in the fields of human interaction and dialogue, and we invite participation from the academic, scientific and religious communities to aid us in the transition to non-violent, and non-killing societies.

Mathematicians and mathematics educators are among those invited to participate in creating the transition to nonviolent and non-killing societies. How do we respond to this appeal?

The State of the World and Mathematics

Mathematician Mikhail Gromov, the 2009 Abel Prize laureate, says:

> Earth will run out of the basic resources, and we cannot predict what will happen after that. We will run out of water, air, soil, rare metals, not to mention oil. Everything will essentially come to an end within fifty years. What will happen after that? I am scared. It may be okay if we find solutions, but if we don't then everything may come to an end very quickly.
>
> Mathematics may help to solve the problem, but if we are not successful, there will not be any mathematics left, I am afraid! (as quoted in Raussen and Skau 2010, p. 401)

I am also afraid. What kind of world are we leaving to the future generations? The future may not be. All our proposals for better educating the future generations may be voided. The tensions within our contemporary societies, both intranational and international, add to the feeling of fright and fear. As mathematicians and mathematics educators we have a responsibility for the future. We have to find ways to both recognize and respond to this responsibility.

A Conference on Sustainable Development (Rio+20), organized by the United Nations in June 4–6, 2012, in Rio de Janeiro, will have the participation of all the disciplines. Mathematics plays a transdisciplinarian role in these discussions. Indeed, mathematics is deeply involved in the interdisciplinary research that needs to take place in preparation for the conference.

Christiane Rousseau, the Vice-President of the Executive Committee of International Mathematics Union (IMU), herself a pure mathematician, announced the endorsement given by IMU to the Conference Rio+20 and to the broad project Mathematics of Planet Earth 2013. This endorsement states:

> Earth is a planet with dynamic processes in the mantle, oceans and atmosphere creating climate, causing natural disasters, and influencing fundamental aspects of life and life-supporting systems. In addition to these natural processes, humans have developed systems of great complexity, including economic and financial systems; the World Wide Web; frameworks for resource management, transportation, and energy production and utilization; health care delivery; and social organizations. Human activity has increased to the point where it influences the global climate, impacts the ability of the planet to feed itself and threatens the stability of these systems. Issues such as climate change, sustainability, man-made disasters, control of diseases and epidemics, management of resources, and global integration have come to the fore. Mathematics plays a key role in these and many other processes affecting Planet Earth, both as a fundamental discipline and as an essential component of multidisciplinary and interdisciplinary research. (see http://www.mpe2013.org)

As educators, we must respond. A number of mathematics educators, including Nicoletta Lanciano of the Department of Mathematics of the Università di Roma "La Sapienza" have developed a project, independently of the IMU project, to involve schools with the major objective of showing the students local specificities in the Planet Earth. The Project Parallel Globe (see http://www.globolocal.net) attempts to recreate, for students, important stages in the development of mathematics, which are related to the observation of the skies. The project intends to assist students in visualizing their position on the Earth surface in relation to the position occupied by other countries, including observing the way the Sun illuminates different regions of the Earth in real time. This visualization helps students to understand time zones and the alternation of the seasons on the planet. This tool of didactic practice is rich in mathematics content, as it allies observation, concrete experimentation with reflection, and data collection. In the development of the project, it became fundamental to share the results from countries of different longitude and latitude, thus helping to clarify the semantic and symbolic differences of the distinctions North-South, top-bottom, over-under, up-down in different languages and cultures.

Intercultural awareness has obvious political implications for a democratic perception of globalization, and it supports respect for difference and the recognition that all nations are part of the same global system, one that is threatened. The main objective of the project is to convey to students the message that civilization is threatened, that all nations share different but common environmental conditions, and that mathematics is an important instrument for monitoring these conditions.

A Challenge to Mathematics Educators and the Preparation of Teachers

A remarkable conference, "Visions in Mathematics—Towards 2000," was held in Tel Aviv, Israel, from August 25 to September 3, 1999. It was an important reunion of leading mathematicians worldwide, gathering on the eve of the twenty-first century to discuss the past and future of mathematics, its importance, and its methods. In this conference, Gromov delivered an address where he pointed to new directions for the development of mathematics, ones resulting from the sociocultural context rather than the conceptual necessities and details intrinsic to established mathematics theories. We need another mathematics. Gromov calls these new mathematical structures "soft," because they consist of greatly flexible hypotheses. These remarkable ideas, although very difficult, clearly indicate that the new generation of scientists, engineers, and, obviously, mathematicians, will need broader attitudes towards mathematics.

The challenging problems require, besides new mathematical techniques, the training of a new generation of researchers in the mathematical sciences. Again, citing Gromov (1998):

> We shall need for this the creation of a new breed of mathematical professionals able to mediate between pure mathematics and applied science. The cross-fertilization of ideas is crucial for the health of the science and mathematics. (p. 847)

All these new considerations are primarily addressed to research mathematicians, but it is undeniable that they pose an even great challenge to mathematics educators. It is questionable if we should insist in keeping, in education, content which is consuming school time and energy instead of moving more rapidly into the new concepts of mathematics, as suggested by Gromov and others. The same question is applicable to the new physics, the new biology, and other scientific fields. It is undeniable that this new face of mathematics is more attractive to students. The digital natives feel that the traditional mathematics that still dominates the curricula is obsolete, boring, and useless. I am convinced that this is the main cause of the bad results in tests.

The New Mathematics

The new mathematics depends, of course, on basic mathematics. But to what extent shall we insist on the basics? Thanks to the amazing technology available, it is possible to accelerate the acquisition of the basic mathematics that is necessary—a small part of what is in the usual programs—and to step, rapidly, into the new mathematics. The basic mathematics includes mainly concepts, not techniques. Curricular development should focus on accelerating the teaching of the things that are effectively basic in traditional mathematics, which are concepts. Instead, much of the time and energy of teachers still goes into insisting on skills.

Mathematics, as a science, has specificities. Steve Kennedy (2003) writes:

> Math is different from the other sciences. In a very real sense the problems, motivations and verification of mathematics come from inside the discipline itself, whereas the other sciences look to the world of phenomena for problems and affirmation. The chemist whose experiment yields a result within six decimal places of his theoretical prediction has good reason to feel pretty pleased with his theorizing. A mathematician rarely finds herself in such an empirically happy place vis-à-vis her theories. Usually a mathematician has only the cold reassurance of logic for comfort; the universe does not deign to validate our work except indirectly, when the work proves useful as a model in another science. (p. 180)

The difficulty is to bridge the gap between the internal advances of mathematics and their utilization. To approximate mathematics to the sciences is to show, in mathematics education, that mathematics is fully integrated with the scientific method, which is an essential component of multidisciplinary and interdisciplinary research. This integration is intrinsic to the proposal of laboratory practices in mathematics education by Eliakim Moore in the beginning of the twentieth century. For instance, Moore (1903) states:

> The boy will be learning to make practical use in his scientific investigations—to be sure, in a naive and elementary way—of the finest mathematical tools which the centuries have forged; that under skilful guidance he will learn to be interested not merely in the achievements of the tools, but in the theory of the tools themselves, and that thus he will ultimately have a feeling towards his mathematics extremely different from that which is now met with only too frequently—a feeling that mathematics is indeed itself a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions. (p. 408)

Rather than proposing a shortcut, Moore proposes restoring mathematics education to the original roots of mathematics development in modernity. The advances proposed since the sixteenth century recognize mathematics as the main support of scientific inquiry.

Some examples of shortcuts for presenting advanced mathematics in a simple and contextual way are the proposals exposed in the books *Calculus Made Easy*, first published in 1910, by Silvanus Thompson, which has been generally repudiated by mathematicians, and *Lectures on Physics*, based on Richard Feynman's lectures in 1961–1963. In both books, authored by distinguished scientists (nonmathematicians who are users of advanced mathematics), content is rapidly presented, with adequate rigor for its purpose. Finding the equilibrium between accessible presentation and acceptable rigor is a major challenge for mathematics educators. The greatest challenge is to perceive these changes, to understand the new, and to develop methods for transmitting this to teachers.

Children must be prepared for a future that we cannot envisage. To prepare children to be proficient in obsolete mathematics is to prepare them to the anguish of being marginal in the future, because they will possess outdated knowledge. Avoiding this anguish is an important feature of social justice. For me, social justice should be understood as an attempt to satisfy the basic needs for a good life: freedom and choice; health and bodily well-being; and good social relations anchored on security, peace of mind, and respect for spiritual experience. We must avoid giving students the illusion that by passing the tests we have now and obtaining good grades they are somehow prepared for the future. This illusion is fallacious and a denial of social justice. The inadequacy of tests is not new. Évariste Galois, more than two hundred years ago, clearly denounced a reliance on tests: "Are you quite happy to do well in the test? Do you believe you will be finally appointed as one of the two hundred geometers that will be admitted? You believe you are prepared: you are mistaken, this is what I will show you in a next letter" (Galois 1831). He died before writing the next letter.

The New Education

Education, in this era of science and technology, challenges the established approaches "validated" by results in standardized tests. The goals of education go much beyond merely preparing for professional success. Education has a responsibility to build up saner attitudes toward the self, toward society, and toward nature. We are primarily faced with preparing teachers to assume a different attitude in their teachings, responding to the new challenges. Educators must be creative.

I believe the key problems in the preparation of teachers of mathematics are related to inadequate visions of the purposes of education and of the role of mathematics teachers as educators. Prospective and in-service teachers of mathematics should be always reflecting about the changes in education that are consequences of profound changes in society, particularly in the demographic scenario, in production, in information, in communication, and in the environment.

Here, I elaborate on the purposes of education as a preliminary to discussing the role of mathematics teachers as educators. I identify a double purpose for why societies establish educational systems:

1. To promote citizenship (which prepares the individual to be integrated and productive in society), which is achieved by transmitting values and showing rights and responsibilities in society.

2. To promote creativity (which leads to progress), which is achieved by helping people to fulfill their potentials and rise to the highest of their capability.

The practice of education is in the present. The major challenge to educators is to manage, in this process, the encounter of the past and of the future; that is, the transmission of values rooted in the past, which leads to citizenship, and the promotion of the new, for an uncertain future, which means creativity. But in this process, we must be careful. We do not want to transmit docile citizenship, where our students accept rules and codes that violate human dignity, and become permanently frightened; instead, we want them to assume a critical attitude toward obedience. Nor do we want to promote irresponsible creativity, where our students become bright scientists creating new instruments to increase inequity, arrogance, and bigotry; we want them to instead be conscious of their acts and of the consequences of their creation. Hence, the goals that I hold important in education, and hence in mathematics education, are—

- the transmission of values rooted in the past, which leads to citizenship, but not docile citizenship; and

- the promotion of the new, for an uncertain future, which means creativity, but not irresponsible creativity.

The transmission of values is intrinsic to cultural encounters. Cultural encounters have very complex dynamics. This encounter occurs between peoples, as occurred in conquests and colonization, and between groups. It also occurs in the encounter between the young man or woman, who has his or her own culture, and the culture of the school, with which the teacher identifies. The so-called civilizing process, carried on by colonizers, is essentially the management of this dynamics. I claim the same occurs in the educational process. Didactics and pedagogy are strategies to manage cultural encounters of students and teachers. Therefore, an important component of mathematics education is to reaffirm and, in many cases, to restore the cultural dignity of children. But a tradition alien to children supports much of the content of current programs. Children are living in a civilization dominated by mathematically based technology and by unprecedented means of information and communication, but schools present an obsolete worldview.

It is equally important to recognize that improving opportunities for employment is a real expectation that students and parents have of school. But preparation for the job market is indeed preparation for the capability of dealing with new challenges. Many careers exist that require different kinds of knowledge and experiences but remain unfilled because of the lack of able candidates. There is a need for change. But what to change and how to change? Ideally, the advances of research in mathematics education produce better-qualified teachers, ones capable of promoting innovative education. But, regrettably, the focus on passing tests dominates school systems and is reinforced by offering teachers rewards, such as salary increases, if their students are successful in the tests. School officials often support this practice, because they are rewarded with grants and other government subsidies. This reward system is a subtle form of corruption, which paves the way to explicit corruption, a flagrant violation of social justice.

Responsible governance should look carefully at the disequilibrium among preparation of graduates and the needs of the job market. Robert Reich (1992), professor of economics at Berkeley and former Secretary of Labor of the Clinton Cabinet, extensively discussed this disequilibrium some years ago.

> Education for all, which is frequently given as a strategy for Social Justice, has many problems and the fact that more and more people are becoming educated, with emphases in science, technology and engineering, sounds like a good thing. It is, indeed, progress. But it is an illusion that "education for all" is the key to economic growth and prosperity and good jobs. We have to analyze the context in which this progress takes place and the fitness and quality of it. There is no point in preparing children for jobs that will probably be extinct when they reach adulthood. (Reich 1992)

Education for all results in an extraordinary amount of people going to school with the hope of finding good jobs. But there are reasons for caution. (For a harsh view of the future of employment and the inadequacy of current educational systems, see Viviane Forrester's 1999 book *The Economic Horror*.) The expansion can dilute the quality of graduates, giving space to less able individuals into the system. Bright students are poorly employed, and the ruthless and often fruitless fight for a permanent job may soon disillusion them. We need more research with the objective of finding out how the labor market will accommodate those who emerge from the school systems. Some results have been reported. But, as of yet, many programs remain firmly attached to the traditional curricula, disregarding the disequilibrium between the preparation of graduates and the needs of the job market.

In 2001, in a seminar at the United Nations Educational, Scientific, and Cultural Organization (UNESCO) Institute for Information Technology in Education, Seymour Papert denounced the enormous amount of resources that are wasted in obsolete education:

> Using computers connected to the Internet students can obtain better and quicker access to sources of historical as well as scientific knowledge; they can explore economics as well as physics by making models and simulations; the rigor of mathematics can be extended to areas that were previously inaccessible. *But in the midst of these explosions of change the institution of School has remained as remarkably constant over time as it is across countries.* So why am I wasting time drawing attention to familiar facts and problems that are already being addressed? The answer is saddening: Although the problem is widely recognized, its depth is seldom appreciated. *Most of those billions of dollars are being wasted.* (Papert 2001, emphasis in original)

Indeed, this waste means that much of the traditional content that exhausts current programs should be drastically changed. It may be a big mistake to insist on mathematics curricula simply because they satisfy criteria of rigor. Some defend that the satisfaction of such criteria are enough to justify content. Curriculum proposals are frequently disguised as new methods to teach the same content, mostly inappropriate and obsolete. Much cost and energy is devoted to showing how to do better at what remains disinteresting, obsolete, and useless, as denounced by Papert (2001). These remarks may be interpreted by many as suggesting a reduction of the importance of mathematical content. This interpretation is grossly mistaken. We need *more and better* mathematical content, but *not the same content*. What I say is that methodological innovation should be directed to making advanced mathematics attractive and teachable. Compromising rigor, in benefit of generating interest and motivation, cannot be interpreted as conceptual errors, or as relaxing the importance of serious mathematics in schools.

Mathematics and Mathematics Education in a Changing Civilization

Mathematics is a fascinating cultural endeavor. It is seen as the imprint of rationality and, indeed, it is the dorsal spine of modern civilization. All the spectacular achievements of science and technology have their bases in mathematics. And the institutions of modern civilization—mainly economics, politics, management, and social order—are rooted in mathematics. It is no surprise that accomplished scholars are devoted to mathematics. A good number of successful citizens who did not accomplish well in mathematics in their school years, and sometimes even failed, put their trust in mathematics in the educational systems.

Administrators, teachers, parents, students, and the population in general, see mathematics along with reading and writing as the principle subjects in schools. But society regards those who do well in mathematics as geniuses, and those who fail are stigmatized. There is a lack of recognition that there are different interests, different creativity, and different talents among different individuals, and particularly among different children. Mathematics acts a selector on intellectual elites. These elites too often pursue the same patterns of society, impregnated with arrogance, inequity, and bigotry, which is a clear violation of social justice.

When looking at mathematics education, we may identify two positions:

1. To use education as a strategy for teaching mathematics (a position defended by the stakeholders previously mentioned).

2. To teach mathematics as a strategy for good education.

Here, I like to use a metaphor. I recognize that the great energy we have in the planet, physical and intellectual and creative, comes from children. Metaphorically, I see children as our Sun. Position 1 sees mathematics presented as a discipline cold and austere. (It was Bertrand Russell who wrote, "Mathematics … possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture.") Position 1 implies children, which are full of energy, like the Sun, revolving around the cold and austere focus of mathematics, metaphorically cold and austere as the Earth. Thus, I call Position 1: the Ptolemaic version of mathematics education.

I, however, fully identify with Position 2. The focus of our mission as educators resides in children and young and elderly adults—in general, those who are the reason and the source of energy for educational action. In this Copernican view, the disciplines, which revolve around those being educated, are merely instruments in this action. Disciplines are, thus, in permanent reformulation, reflecting social and cultural contexts and the queries, wishes, and needs of those being educated. Is this a good strategy for a good education? I believe so!

We have to look into history and epistemology with a broader view. The denial and exclusion of the cultures of the periphery, so common in the colonial process, still prevails in modern society. The denial of knowledge that affects populations is of the same nature as the denial of knowledge to individuals, particularly children. To propose directions to counteract ingrained practices is the major challenge of educators, particularly mathematics educators. Large sectors of the population do not have access to full citizenship. Some do not even have access to the basic needs for survival. This limited access is the situation in most of the world, and it occurs even in the most developed and richest nations. (Further discussion about these matters is the objective of ethnomathematics, which is not discussed here; see D'Ambrosio 2006.)

A new world order is urgently needed. Our hopes for the future depend on learning—critically—the lessons of the past. When we look at the history of mathematics since the early mathematical manifestations of man (and woman), we recognize the development of techniques to compare, to classify and to organize, to measure and to count, to infer and to conclude, much before mathematics is formalized. We also recognize mathematical ideas in the confluence of various modes of understanding, such as the religions, the arts, the techniques, the sciences. We must assume a *transdisciplinarian* posture, and we need to look at all development and modes of understanding in different cultural environments, in different traditions—that is, we must assume a *transcultural* posture. This new posture may restore to mathematics its characteristic of being the *most universal mode of thought* and may allow it to face the *most universal problem facing humanity*, which is survival with dignity.

The enormous changes in society, particularly due to demographic dynamics, have raised the exclusion of large sectors of the population, both in developed and undeveloped nations, to unbearable levels. The exclusion of countries from the benefits of progress and advancement is unsustainable. Any explanation for the current perverse concept of civilization requires a deep reflection on colonialism. This reflection should not aim at blaming one group or another and should not be an attempt to redo the past. Rather, it is the moment to understand the past as a first step to move into the future. Because mathematics has everything to do with the State of the World, its autonomy in the curriculum, and its central role as the dominating discipline and as an educational sphere in itself, should be reconsidered. Paraphrasing Gromov (1998), we shall need for this the creation of a new breed of mathematical teachers, able to mediate between mathematics and the other disciplines. But current curricula, in all levels of education, look like a selection of non-overlapping sets. Each discipline has its own domain. As a result, there is a lack of perception among teachers of the relation of mathematics to a broader vision of the world and of society.

Curriculum is the strategy for the educational action. Educational action should offer three instruments that, together, provide what is essential for citizenship in a world moving swiftly toward a planetary civilization. These instruments are the communicative instruments, the analytic/symbolic instruments, and the technological instruments. They constitute the modern *trivium*, which I call respectively *literacy*, *matheracy*, and *technoracy* (D'Ambrosio 1999). This *trivium* is a proposal for a curriculum based on developing a broad perception of the complexity of the world and of society and providing the instruments to deal with such complexity.

Literacy is the critical capability of processing information, such as the use of written and spoken language, of signs and gestures, of codes and numbers. Nowadays, reading must also include the competency of numeracy, of interpretation of graphs and tables, and of the other means of informing the individual. Reading even includes understanding the condensed language of codes. These competencies have much more to do with screens and keys than with pencil and paper.

Matheracy is the critical capability of inferring, proposing hypotheses, and drawing conclusions from data. It is a first step toward an intellectual posture, which is almost completely absent in our school systems. Matheracy is closer to the way mathematics was present both in classical Greece and in indigenous cultures. The concern goes much beyond counting and measuring. Matheracy proposes a deep reflection about humans and society and aims at explaining and understanding reality. It is, indeed, symbolic analysis. This is the central idea behind the origins of mathematics. This competency should not be restricted to an elite, as it has been in the past. It is not the result of appropriation of skills but is instead acquired through competency to analyze.

Technoracy is the critical familiarity with technology. Of course, the operative aspects of it are, in most cases, inaccessible to the lay individual. But the basic ideas behind technological devices, their possibilities and dangers, and the morality supporting the use of technology, are essential issues to be raised among children at a very early age. As a historian, my resource is the critical perception of past and of future as a guide for action in the present, and history show us that ethics and values are intimately related to technological progress. Proficiency in mathematics means much more than counting, measuring, sorting, comparing, and solving problems aimed at drilling. Regrettably, even conceding that problem solving, modeling, and projects are practiced in some mathematics classrooms, the main importance is usually given to developing skills, particularly in the manipulation of numbers and operations. But problems and situations present in daily life are new and unexpected. Students should be prepared to tackle the new. The three instruments together, which obviously include reading, writing, and basic mathematics, constitute what is essential for citizenship in a world moving swiftly toward a planetary civilization.

Concluding Remarks

Civilization, as well as life of all the animal species, is threatened. There will not be, as we are told in the *Epic of Gilgamesh* or in the biblical episode of Noah, a privileged group of humans that will survive. I understand the threat to the species as the broadest violation of social justice. I tried to avoid, in this concluding chapter, commenting on or reinforcing the proposals of the previous chapters. All are written with extreme competence, presenting improvements of mathematics education aimed at social justice, something essential for citizenship. My objective in writing this concluding chapter was to bring to the attention of mathematics educators the need to give their thoughtful and serious consideration to a broader concept of social justice, focusing on the State of the World and the real threat to civilization. Paraphrasing Bertrand Russell and Albert Einstein in the 1955 Pugwash Manifesto (http://www.pugwash.org/about/manifesto.htm), a *New Thinking* is needed to achieve social justice, meaning equilibrium and safety, in a world menaced by exhaustion of resources, which leads to war and fear. Mathematicians and mathematics educators have powerful means of developing new concepts and techniques to cope with the major threats to the survival of civilization.

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