

The game Closes the Box: a proposal for teaching Probability

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Abstract: Mathematical games play a significant role in teaching mathematics and statistics, as they offer a practical and engaging approach to learning often complex statistical concepts, such as the study of probability. Given this scenario, this article aims to present and discuss the potential of using the game *Close the Box* to teach notions of probability. This is qualitative research that was developed in a class made up of 25 students in the 2nd year of elementary school at a public school in São Paulo. The results show that students understood some basic aspects, such as random, unlikely, and very likely events, and developed signs of probabilistic literacy. In short, we conclude that working with games provides students with fun and stimulating moments and can help them to understand, in an informal way, probabilistic aspects necessary for decision-making.

Keywords: Games in Teaching. Statistical Education. Elementary School. Probability.

El juego Cierra la Caja: una propuesta para enseñar Probabilidad

Resumen: Los juegos matemáticos desempeñan un papel importante en la enseñanza de matemáticas y estadística, ya que ofrecen un enfoque práctico y atractivo para aprender conceptos estadísticos a menudo complejos, como el estudio de la probabilidad. Ante este escenario, este artículo tiene como objetivo presentar y discutir el potencial del uso del juego, denominado *Cerrar la Caja*, para la enseñanza de nociones de probabilidad. Se trata de una investigación cualitativa que se desarrolló en una clase compuesta por 25 estudiantes del 2º año de la escuela primaria de una escuela pública de São Paulo. Los resultados muestran que los estudiantes comprendieron algunos aspectos básicos como eventos aleatorios, improbables y muy probables y desarrollaron signos de alfabetización probabilística. En definitiva, concluimos que trabajar con juegos proporciona a los estudiantes momentos divertidos y estimulantes y puede ayudarles a comprender de forma informal aspectos probabilísticos necesarios para la toma de decisiones.

Palabras clave: Juegos en la Enseñanza. Educación Estadística. Enseñanza Primaria. Probabilidad.

O jogo Fecha a Caixa: uma proposta para o ensino de Probabilidade

Resumo: Os jogos matemáticos desempenham um papel significativo no ensino de Matemática

e Estatística, proporcionando uma abordagem prática e envolvente para aprender conceitos, muitas das vezes complexos, como o estudo da Probabilidade. Diante desse cenário, este artigo tem o objetivo de apresentar e discutir as potencialidades do uso do jogo *Fecha a Caixa* para o ensino de noções de Probabilidade. Trata-se de uma pesquisa de cunho qualitativo, desenvolvida em uma turma composta por 25 alunos do 2º ano do Ensino Fundamental de uma escola pública de São Paulo. Os resultados mostram que os alunos compreenderam alguns aspectos básicos, como eventos aleatórios, pouco provável e muito provável e desenvolveram indícios de letramento probabilístico. Em suma, concluímos que o trabalho com jogos proporciona momentos divertidos e estimulantes aos alunos, podendo auxiliá-los na compreensão informal de aspectos probabilísticos necessários para a tomada de decisão.

Palavras-chave: Jogos no Ensino. Educação Estatística. Ensino Fundamental. Probabilidade.

1 Introduction

Probability is an essential subject for dealing with uncertainty and for predicting outcomes in various situations, be they economic, social, or political. In the educational context, the teaching of probability plays a crucial role in enabling students to understand and apply probabilistic concepts in their daily lives so that they can exercise their citizenship in a society that increasingly requires citizens to have basic knowledge of statistics and probability. Thus, the teaching of probability topics in basic education has become as necessary as reading and writing in this scenario.

The formal teaching of probability and statistics involves a lot of theoretical tools and complex language, which does not always allow students to effectively learn the content proposed within these topics. According to Lopes (2010), mathematics curricula should have interconnected structures when it comes to teaching probability and statistics, prioritizing context. However, it is common to find students who are unmotivated by both mathematics and statistics in the classroom.

An alternative way to present aspects of probability and to motivate students to learn statistical content is to use games that make the class more interesting. In this context, the collaborative environment stands out in relation to aspects of solitary and passive learning, where the teacher is the center of attention and the students are mere spectators. Therefore, one of the aims of this article is to provide new strategies for students to learn about the benefits and power of statistics and, in particular, probability.

The difficulties identified for the subject of mathematics also extend to the content and concepts related to probability. Probability, the branch of statistics that creates, develops, and researches models to study random experiments or phenomena, often causes discomfort among both students and teachers. The teaching of probability is recommended from the early years of elementary school, since the knowledge and skills acquired over the years contribute to the formation of the student as a critical and aware citizen.

The use of games as a methodological teaching strategy provides children with a playful and enjoyable environment, breaking the paradigm of teaching based on memorization of steps, rules, and decontextualized techniques. In light of the above, the aim of this article is to present and discuss the potential of using the game *Close the Box* to teach probability concepts to 2nd grade students in a public school in the state of São Paulo. In the following sections, we present the theoretical framework, the methodology used, the results obtained, and the final considerations.

2 Theoretical framework

Recently (2019-2022), Brazil has faced the challenges posed by the Covid-19 pandemic. Measures have been taken to prevent and contain the spread of the virus, which is spreading rapidly. In this context, the ability to predict the behavior of the transmission of the disease in the coming days, weeks, and months has become essential to making decisions based on probabilistic data. In this sense, mastering skills related to probabilistic thinking from an early age is urgent and necessary. Mastering these skills can enable us to make predictions and decisions even before they occur.

It is essential to introduce the study of probability in the early years of primary school, so that children, adolescents, and adults learn to make decisions based on data from their reality and to contribute as citizens of the world, in the world. In this direction, the National Common Curriculum Base — BNCC (Brazil, 2017) provides the following guidelines:

With regard to the study of notions of Probability, the aim in Primary School - Early Years, is to promote understanding that not all phenomena are deterministic. To this end, the beginning of the proposal to work with probability is centered on developing the notion of randomness so that students understand that there are certain events, impossible events, and probable events (Brasil, 2017, p. 270).

Regarding the Probability skills expected in the second year of primary school, the following skill is added: “(EF02MA21) Classify the consequences of random everyday events as *unlikely*, *very likely*, *improbable* and *impossible*” (Brazil, 2017, p. 285).

Batanero (2015) points out that teaching probability, when based on situations in children's reality, allows them to experience random events, develop their own language to describe their observations, relate their previous intuitions to the data from the experiment, and make connections between probability and other subjects, both mathematical and non-mathematical.

From this perspective, we understand that children's engagement with probability concepts is more expressive when they experience situations of uncertainty based on problematization rather than on rules or definitions. It is during problematization that they analyze, reflect, and discuss their previous intuitions with their classmates and question the data revealed. Meanwhile, it becomes necessary and indispensable for the children to verbally express the results that could have occurred in comparison with what actually happened, thus beginning the construction of concepts related to uncertain events.

Given the importance of studying probability from the early years of primary school, it is common in Brazil that many teachers are still not convinced of its relevance, or even consider the inclusion of this subject as something completely inappropriate and unnecessary in the school curriculum (Campos & Pietropaolo, 2013). On the other hand, Lopes (2008, p. 100) reports that “given the rapidly changing world in which we live, knowledge of the probability of events is essential in order to speed up decision-making and make predictions” from the moment children enter school.

However, developing a pedagogical proposal aimed at teaching and learning probability through a game allows children to experience and investigate the frequencies of the results of random experiments. This makes it possible to explore the probabilities of these results through interpretation, reflection, and discussion with peers, allowing the development of critical analysis and argumentation.

3 Methodology

With the aim of presenting and discussing the potential of using the Close the Box game to teach probability, this study is part of a qualitative approach. According to Minayo (1995), qualitative research corresponds to a more intense space with regard to the relationships of processes and phenomena that are not reduced to the operationalization of variables. It is a research that works with a universe of meanings.

It also belongs to the interpretive paradigm because it involves direct contact between the researchers and a deep and intense experience with the participants. In other words, "researchers make an interpretation of what they see, hear, and understand. Their interpretations cannot be separated from their backgrounds, histories, contexts, and prior understandings" (Creswell, 2010, p. 209).

Based on a qualitative, interpretive approach, the research was conducted by a teacher with a class of 25 students in the second year of primary school in a municipal school located in the eastern zone of the city of São Paulo. When considering the structure of the data collection process, we chose to use videos, recognizing their importance and flexibility as a resource for collecting oral and visual information. Videos are able to capture significant behaviors and puzzling interactions, allowing researchers to review the data on an ongoing basis (Powell, Francisco & Maher, 2004).

We have been inspired by the Lesson Study methodology to structure and develop our study. This methodology originated in Japan at the end of the 19th century and has been adopted in various countries in the West and East, with adaptations to the particularities of local educational systems and cultures. The methodology, mediated by experienced researchers, aims to improve student learning and promote the professional development of teachers. This approach unfolds in three main stages, which include lesson planning (collective and individual), implementation of the planned lessons, and finally reflection on the lessons taught, which can lead to replanning of future lessons.

In this study, the training methodology was guided by the original three stages of the Lesson Study and incorporated the *training of trainers* and *dissemination of results* stages proposed by Martins (2020) in his doctoral thesis. The *training of trainers* stage involved the trainers (researchers in this research) and consisted of an in-depth study of theories that discuss notions of probability, in addition to holding debates on how the subject is proposed in the prescribed and presented curriculum of the São Paulo Municipal Network, as well as exploring the approach that the teacher could follow in developing her lesson. As for the *dissemination of results* phase, the publication of the article is an integral part of this phase of the process.

In the planning phase, we invited the participating teacher, who is also the fourth author of this article, to collaborate in planning her lesson. Because of the need for coordination between the trainers, who live in different states, and the convenience of recording the session so that the teacher could revisit the agreements made, we decided to conduct the planning synchronously using the Zoom application.

We noticed that during the planning phase, an interesting discussion arose about the use of games in the math classroom. Our concern was the importance of presenting the Close the Box game to students without losing sight of the mathematical concepts behind it.

In this way, we planned the lesson with great intentionality, taking into account the *moments* of pedagogical intervention proposed by Grando (2007). These moments are explained along with the description and analysis of the lesson in the next section.

4 The Close the Box game and the necessary adaptations

This game fulfills the learning and development objectives “(EF02M22) Classify the outcomes of random everyday events as *unlikely*, *very likely*, *unlikely* and *impossible*” and “(EF02M32) Play strategy games in which the goal is to find a *way* to win and justify the decision of the *way* taken” prescribed in the City Mathematics Curriculum (São Paulo, 2017, pp. 91-92).

The São Paulo Municipal Network's own curriculum document encourages the use of games in the classroom, pointing to them as an important teaching strategy that promotes better acquisition of mathematical knowledge through playful activities. These activities are included in the right to learn, both as an object of knowledge in themselves and as a skill to be developed in the curriculum.

Thus, as mentioned above, we carried out a study with students in the second year of primary school, using the game *Fechar a Caixa* (Close the Box), with the aim of informally introducing the concepts of probability.

Of Dutch origin, Close the Box is a game that was widely used by sailors throughout Europe, as it allowed for cash bets. Essentially, the object of the game is to close all the numbers in a wooden box (from 1 to 9) by rolling two six-sided dice numbered 1 to 6. The player can use the value of each die or the sum of the two dice to complete any combination of the numbered tiles. The player continues to play as long as new rolls and combinations allow him to close the remaining open spaces. At the end of the game, the scores of the open boards are added up and the number of points lost by each player is recorded. The winner is the player with the fewest points lost.

In our study, in an attempt to introduce the notion of probability, which considers random everyday events as *unlikely*, *very likely*, *improbable* and *impossible*, we adapted the game by deliberately including the cards 10, 11, 12, and 13. The inclusion of these cards is justified because the sum of 12 is the limit, since we only have two dice, each with six faces. Having explained the game, we will now present the research procedure.

5 Results

The planned activity for the 2nd grade was conducted by the teacher on June 26, 2023, in the morning. There were 25 students present, out of 29 in the class, and a teacher who observed and videotaped the lesson. It should be noted that the photographs were taken from the videos.

We started with the first step suggested by Grando (2007), called *familiarization with the game material*, which consists of getting the students in touch with the components of the game. In the case of Close the Box, this material would be the dice and the cards numbered from 1 to 13.

In this first moment, the teacher explained to the students that they would be doing an activity involving a game that sailors often played as a *pastime*. She also highlighted some of the skills that would be practiced during the game, such as mental arithmetic. She then asked the students if they knew what this type of calculation was. The answers were quite similar, including *using your brain during the game*, *using your mind*, and *doing mental calculations because we have to think*, among others.

Based on this response, the teacher asked them how they use mental arithmetic to solve a given operation. One of the students mentioned simple counting in 3s, while another explained

Teacher, when I count $7 + 3$, I put the 7 on my head [points to his head] and raise 3 fingers and count 8, 9, and 10 [lowers fingers as he counts], and the result is 10!

Figure 1 illustrates this moment.

Figure 1: Initial conversation about the Close the Box game

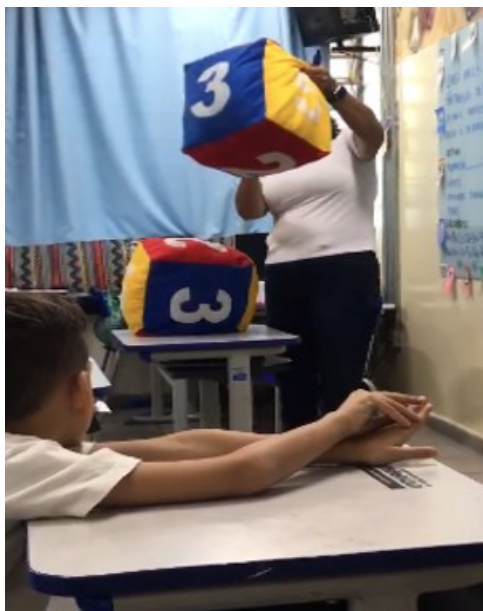


Source: Research data.

From this initial moment, the teacher explains to the students the materials that will be used to make Close the Box Game, initially presents the data, and asks questions:

How many sides does a dice have? The two dice we are going to use are numbered; what numbers appear on them?

Figure 2: Moment of application of the Close the Box game



Source: Research data.

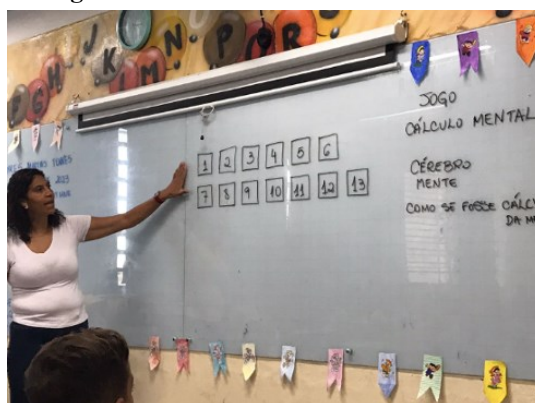
It's worth noting that some students misidentified the number of sides on one of the dice. As a result, the teacher invited some students to come to the front of the class and start handling the dice so that they could identify the number of sides, as shown in the picture below:

Figure 3: Application of the Close the Box Game

Source: Research data.

The teacher then suggested that two students go to the front of the class and begin to explore the dice, informally introducing the rules of the game. The teacher then suggested that one of them roll the two dice and the other mentally calculate the sum of the numbers obtained. It's worth noting that, at this point, the students had no difficulty mentally calculating the numbers they rolled.

The teacher then explained that in addition to the dice, cards numbered from 1 to 13 would be used to represent the cards on the board. Meanwhile, the teacher began the *rule recognition* moment proposed by Grandó (2007). According to the author, the recognition of the rules of the game by the students can be done in different ways: reading and explaining them by the teacher of the action or identifying them by playing model games, in which the teacher of the action can play games with one of the students, while the rest try to observe the regularities in the moves and identify the rules of the game.

Figure 4: The first results in the classroom

Source: Research data.

As we can see in the picture, the teacher explains the rules of Close the Box, showing that after throwing the two dice, the students should mentally calculate the sum of the faces and select one or two cards to turn over that express the result of the sum of the dice. The teacher gave an example of rolling the dice with the faces 4 and 5, the sum of which was 9. She then explained the possibilities of the cards to be turned over, i.e., cards 7 and 2; 8 and 1; 6 and 3; and 9 itself, among others.

The teacher asked:

And if I roll the dice and the faces 3 and 3 come up, is it possible to turn two cards 3?

The students immediately replied that no two cards had the same number, unlike the dice. However, it was possible to turn over cards 4 and 2 if they hadn't been turned over in previous moves. At this point, the teacher realized that the students had already grasped the rules.

The teacher then encouraged the students to look for ways to turn over the faces (3 and 3) from the result 6. So, in an attempt to systematize this moment in the lesson, the teacher proposed some challenges for the class, such as considering throwing the faces 5 and 5, resulting in 10. At this point, the students were encouraged to think about the possibilities, while the teacher recorded the solutions that emerged on the blackboard.

Figure 5: Moment of reflection on the search for other possibilities



Source: Research data.

The teacher then thought it appropriate to roll the dice again and start a game with two students who had volunteered. The dice were rolled, showing faces 3 and 4, and coming up with 7. The teacher asked, *Which cards can I turn over?* She explained that they should only choose one, so the students agreed and indicated cards 3 and 4.

The dice were then rolled again, coming up 3 and 1. The teacher asked the students to indicate which card or cards should be turned over. One of the students in the watching class said 4. At this point, the teacher made it clear that the flipped cards could no longer be used and that the students should try other possibilities. After several rounds, the couple rolled the dice and found that there were no more cards to turn over. The teacher explained that the game was over and that the students should add up all the numbers on the remaining cards to see how many points they had lost.

It is worth noting that at this point none of the students realized that it was impossible to close card 13 because none of the dice would add up to 13. The teacher then moved on to the third stage proposed by Grando (2007), known as *playing for the sake of playing*. The author points out the importance of playing in order to spontaneously internalize the rules. It is at this point in the game that the mathematical concepts contained in Close the Box are explored. Therefore, it was expected that at this point the students would realize the impossibility of turning over card 13, as they would all be truly experiencing *the game for the sake of the game*.

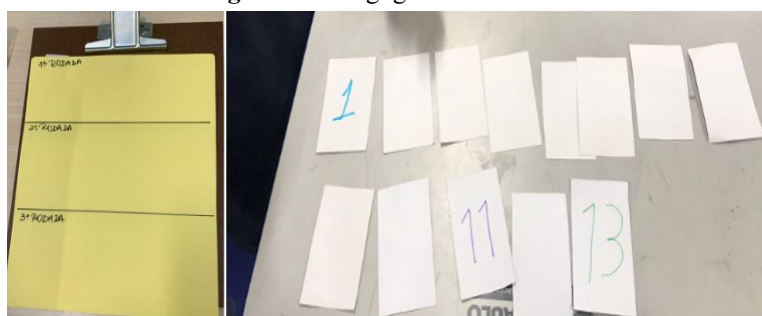
In the *game for the sake of the game* moment, the students were divided into seven trios and two doubles. The teacher then distributed cards numbered 1 through 13 and two dice to each group. While distributing the cards, the teacher briefly went over the rules of the game. At this stage, the teacher lets the students play freely without any instructions.

Figure 6: Moment of the game activity

Source: Research data.

After the *game for the game* moment, the teacher suggested that the students start a new stage by giving each group a sheet of paper. The idea was for the students to record the points lost by each member of the group in each round. The teacher encouraged the students to rely on the written record, especially those who had difficulties with calculations involving larger numbers. This new stage of the game is called *recording the game* as suggested by Grando (2007).

In addition, the students began to play with the pedagogical intervention of the teacher. According to Grando (2007), in this phase, the intention is to provoke the students to analyze their moves, such as predicting the next moves, analyzing possible strategies to be adopted, and identifying *wrong moves*, among other aspects.

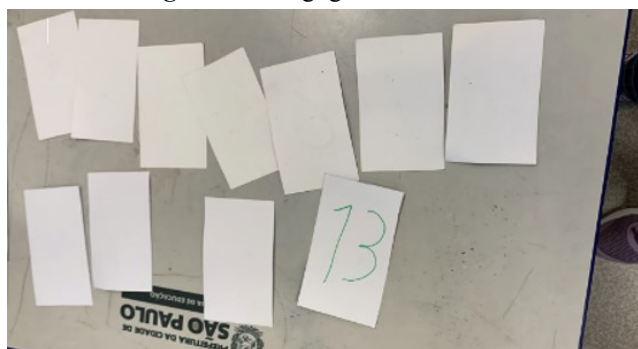
Figure 7: Pedagogical intervention

Source: Research data.

It was an exquisite moment when the students began to play competently, thinking about their moves and the possibilities of future moves. So the teacher circulated among the groups to see if the students were following the rules and if they realized that it was an *impossible* event to turn over card 13, since the greater sum of the two dice would be 12.

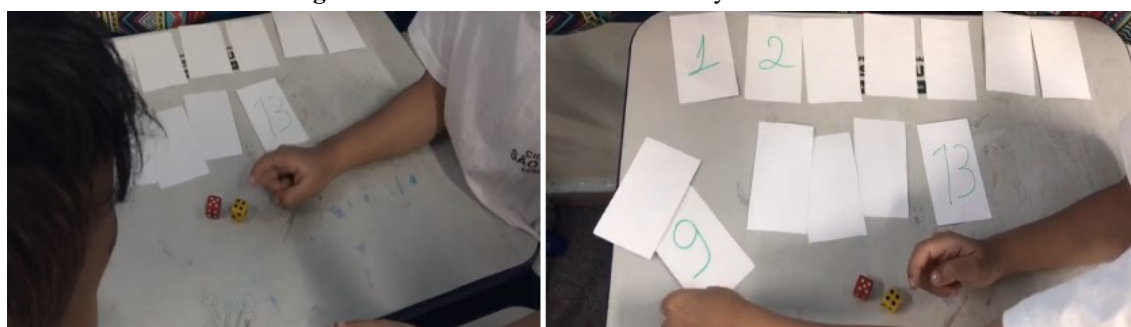
In the first round, three trios realized that card 13 could not be turned over. The words of student Daniel, one of the first to realize this, illustrate this observation: *It's just that the results, they're 6. Six plus six is twelve, so you can only go up to 12.* Another student added, mentioning a possible *flaw* in the game's rules, and asked:

When the dice are rolled, they don't come up to 13. I've played before and they only give up to 12. Then I thought, 6 plus 6 is 12, and there's no way we can get 13. The biggest number on the dice is 6, right? To get thirteen, I'd have to add one more [referring to adding the number one: $6 + 6 + 1$], but I'd need another dice. That's no good!

Figure 8: Pedagogical intervention

Source: Research data.

Another trio, in particular, insisted on the possibility of turning over card 13. One of the students asked the teacher for support and commented: *All we need is the number thirteen; we can't get the number thirteen; we don't even know what number thirteen is on the dice!*

Figure 9: Moment of Data Collection by the students

Source: Research data

The teacher then began a collective discussion with the class, writing the terms *unlikely*, *very likely*, *unlikely*, and *impossible* on the board. The teacher then asked the students about the random everyday events listed on the board. In the first moment of this stage, the students were not yet able to establish a relationship with card 13, but some of them pointed out some events that were impossible to happen, such as, in the words of student Leticia, *seeing a dinosaur alive*. Another student added: *to see a unicorn*.

Some students contributed and showed understanding of what was an unlikely event, such as *the time to stop* and *seeing an elephant on the beach*.

The teacher continued the discussion and provoked the students by manipulating one of the dice, followed by the question: *Is it possible for me to roll the dice and have the number zero come up?* (Figure 10).

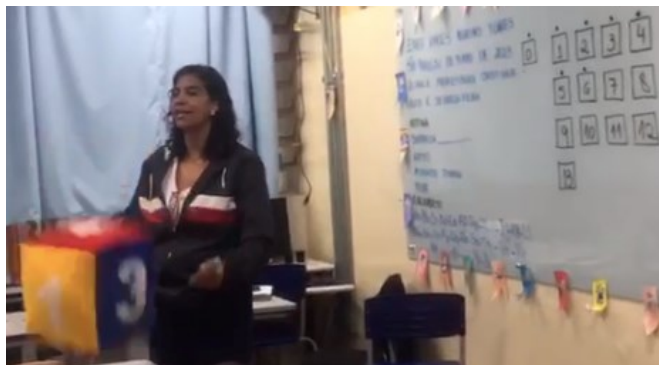
In agreement, the students mentioned that it wasn't possible, and one of them said: *It's unlikely, teacher*.

At this point, the teacher watching the filming asked the students what they thought were *very likely* events. One of the students commented: *Play when you get home from school*.

From these statements, it could be seen that the students were already making connections with random everyday events. In this sense, the teacher informally suggested that they relate these events to the situations in the Close the Box game that they had recently experienced. The teacher asked the class which cards they hadn't been able to turn over and noted them on the board. It turned out that all the groups, without exception, hadn't managed to

turn over card 13, and most of them hadn't managed to turn over card 12 either.

Figure 10: Socializing the results obtained during the game



Source: Research data.

At this point, the teacher asked the students to reflect together. As a result, they identified *turning over card 13* as an impossible event; *the face 6 falling on both dice* and *the same numbers falling twice on the previous roll* as unlikely events; and *turning over cards 1 and 2* as very likely. After this discussion, the teacher systematized the students' knowledge of the concepts of probability, making connections with the game Close the Box, which had been well explored with the children.

After developing the lesson, the teacher watched the footage with the researchers, who were the designated trainers, and redesigned the lesson using Close the Box, now thinking about introducing the concepts of probability in a formal way to try to develop probabilistic thinking.

6 Some final considerations

In this article, we present and discuss the potential of using the Close the Box game to teach Probability notions. We observed that the teacher's practice was re-dimensioned and re-signified based on the considerations made. From the students' discussions, we noticed that the game helped them learn how to classify the results of random events as *unlikely*, *very likely*, *improbable* and *impossible*.

The pedagogical proposal aimed to teach and learn Probability through a game that allowed children to experience and investigate the frequencies of the results of a random experiment. This allowed them to explore the various possibilities of these results through analysis, reflection, and discussion with their classmates, promoting the development of critical analysis and argumentation. The use of the game provided investigative learning based on data analysis, reflection, and discussion, making the environment fun and enjoyable.

From the teacher's practice, it was possible to understand that she is a reflective and problematizing human being, capable of producing pedagogical knowledge in the classroom. She sees herself as a person in constant development, seeking to improve professionally throughout her life. Her approach to creative insubordination allowed her students to develop skills related to Probability (Freire, 2020).

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