



## Being, seeing and recognizing oneself as a teacher who teaches Mathematics: (re)configuring Professional Identities from the perspective of Probabilistic Education


**Lemerton Matos Nogueira**


Universidade de Pernambuco  
Petrolina, PE — Brasil  
✉ [lemerton.nogueira@upe.br](mailto:lemerton.nogueira@upe.br)  
 0000-0001-9841-898X

**José Ivanildo Felisberto de Carvalho**

Universidade Federal de Pernambuco  
Caruaru, PE — Brasil  
✉ [ivanildo.carvalho@ufpe.br](mailto:ivanildo.carvalho@ufpe.br)  
 0000-0003-3981-4805



2238-0345 

10.37001/ripem.v14i3.3831 

Received • 08/03/2024  
Approved • 28/05/2024  
Published • 20/08/2024

Editor • Gilberto Januario 

**Abstract:** This article aims to understand how actions and practices based on studies on Probability Education contribute to the (re)configuration's movement of the Professional Identity (PI) of Teachers who Teach Mathematics (TTM). The study is based on a qualitative and interpretative paradigm, relied on a Research-Training path developed in a study group with 11 TTM. To this end, we are based on a characterization of TTM's PI, which points out a set of elements (macrodomains) that allow us to understand them in their ways of being, seeing and recognizing themselves as such. The results and discussion were based on the apprehension of four Meaning Cores, which indicate that the TTM's PI were forged from the Learning Paths (re)constituted in the collective/collaborative context of the group and interwoven by elements of personal, emotional, moral and sociopolitical nature that emerged from the enterprise of teaching and learning Probability.

**Keywords:** Professional Identity. Probabilistic Education. Teachers who Teach Mathematics. Research-Training.

### Ser, verse y reconocerse como docente que enseña Matemáticas: (re)configuring Professional Identities from the perspective of Probabilistic Education

**Resumen:** Este trabajo tiene como objetivo comprender cómo acciones y prácticas basadas en estudios sobre Educación en Probabilidad contribuyen al movimiento de (re)configuración de la Identidad Profesional (IP) de los Profesores de Enseñanza de Matemáticas (PEM). El estudio se fundamenta en un paradigma cualitativo e interpretativo, basado en un itinerario de Investigación-Formación desarrollado en un grupo de estudio con 11 PEM. Para ello, nos basamos en una caracterización de los PEM IP, que señala un conjunto de elementos (macrodomains) que permiten comprenderlos en sus formas de ser, verse y reconocerse como Tales Los resultados y la discusión se basaron en la aprehensión de cuatro Núcleos de Significado, que indican que los PI PEM se forjaron a partir de las Trayectorias de Aprendizaje (re) constituidas en el contexto colectivo/colaborativo del grupo y entretejidas por elementos de carácter personal, emocional, moral y social. carácter sociopolítico que surgió de la empresa de enseñanza y aprendizaje Probabilidad.

**Palabras clave:** Identidad Profesional. Probabilistic Education. Professors who Enseñan Mathematicians. Investigación-Formación.

### Ser, ver-se e reconhecer-se como um professor(a) que ensina Matemática:

## (re)configurando Identidades Profissionais na perspectiva da Educação Probabilística

**Resumo:** Este trabalho objetiva compreender como ações e práticas pautadas nos estudos sobre a Educação Probabilística contribuem para o movimento de (re)configuração da Identidade Profissional (IP) de Professores(as) que Ensinam Matemática (PEM). O estudo é pautado no paradigma qualitativo e interpretativo, assente em um percurso de Pesquisa-Formação desenvolvido em um grupo de estudos com 11 PEM. Para tanto, nos assentamos em uma caracterização da IP de PEM, a qual aponta um conjunto de elementos (macrodomínios) que nos permite compreendê-los em seus modos de ser, ver-se e reconhecer-se como tal. Os resultados e discussão se pautaram na apreensão de quatro Núcleos de Significação, os quais indiciam que as IP dos PEM se forjaram a partir das Trajetórias de Aprendizagem (re)constituídas no contexto coletivo/colaborativo do grupo e entre(tecidas) por elementos de natureza pessoal, emocional, moral e sociopolítica que emergiram do empreendimento de ensinar e aprender Probabilidade.

**Palavras-chave:** Identidade Profissional. Ensino de Probabilidade. Professores(as) que Ensinam Matemática. Pesquisa-Formação.

### 1 Introduction

The discourses on Professional Identity (PI) — and particularly the Professional Identity of Teachers who Teach Mathematics (TTM) — have gained prominence in the specialized literature and demarcated their academic and scientific relevance, notably for the construction of educational programs and processes (Skott, 2018). In this sense, valuing actions and practices that promote the movement of (re)configuring the PI, especially those related to the personal, professional and inter-relational contexts of teaching, as well as instigating robust discussions that can be built (De Paula & Cyrino, 2020), makes it possible to resist counterproductive formative perspectives and to meet the demands and challenges of the current educational context more effectively.

That said, in this article we have chosen to use studies on Probabilistic Education as a magnifying glass capable of broadening our view of understanding the phenomenon of TTM's PI in Research-Training contexts. This is an important investigative niche that fosters comprehending the links between this construct and the learning of specific mathematical topics in initial and/or continuing education (Graven & Heyd- Metzuyanım, 2019).

We aim to understand how actions and practices based on research on Probabilistic Education contribute to the (re)configuration's movement of the TTM's PI, who participate in a study group on this subject. This objective covers part of the study carried out in the thesis of the first author, supervised by the second. We emphasize some constitutive aspects of the TTM's PI, as well as how they were established and (trans)formed in the context of the group. We analyzed and discussed the results from the constitution of four Meaning Cores (MC) (Aguiar, Aranha & Soares, 2021), which seized evidence of the Learning Paths constituted by the TTM individually and collectively, using as theoretical and epistemological support a set of macrodomains that characterize the TTM's PI.

Initially, we will discuss the theoretical and epistemological perspectives on TTM's PI that support this study, then we will present a few additions from the field of Probabilistic Education.

Next, we present the methodological aspects, the results and the discussions based on four CS that emerged from the analyzes. Finally, as final considerations, we look back at the

main aspects inherent in the group's actions and practices that revealed movements towards (re)configuring PI from the perspective of Probabilistic Education.

## 2 The TTM's PI: a web of interrelated macro-domains

In the context of teaching exercise, identity constructions are exchanged by the various collective spaces where teachers work and construct themselves socially. These include the schools in which they work, the formal and informal spaces of continuing education in interaction with their students, co-workers and the construction of practices related to their disciplinary field. Thus, developing a Professional Identity as TTM is different from developing a teaching identity in other disciplines, since they are unique in terms of the elements that identify them (Levy, Manfredo & Gonçalves, 2012). In this way, the TTM take on ethical, moral and political commitments based on the functions and objectives of their disciplinary field, as well as present idiosyncrasies that go back to their life stories, training and professional development.

We conceive the TTM's PI as constituted/(re)configured in a movement that is essentially continuous, dynamic, temporal and experiential (Cyrino, 2021; De Paula and Cyrino, 2020). Therefore, in addition to the specific knowledge required for teaching as TTM, it is necessary to consider, in an inseparable way, other important domains that characterize them in their actions and practices, and that identify them with each other (Cyrino, 2018). These domains emerge from TTM's biography, from many training experiences and (future) professional performance.

So, Cyrino (2021) assumes a characterization to analyze the constitution's movement of the TTM's PI in collaborative training spaces (initial and continuous training), asserting that such movement occurs from “a set of beliefs/conceptions of the teacher in training interconnected to his self-knowledge, his emotions and knowledge about his profession, associated with autonomy (vulnerability and sense of agency) and political commitment” (Cyrino, 2021, p. 4, author's emphasis).

This characterization represents an important advance in the field of TTM Research Training and in the discussions on the subject in Brazil. It contradicts the traditional canons of understanding the construction of these teachers' identities only in terms of learning the professional knowledge necessary for their practices, beliefs and conceptions. Cyrino's (2017, 2018, 2021) studies on TTM's PI are based on the PI constructs defended by some international researchers with a long tradition of research in this field (Kelchtermans, 2005; Lasky, 2005). In these studies, there is a consensus that issues of a cognitive, affective, emotional, moral and political nature also play a role in the PI's constitution.

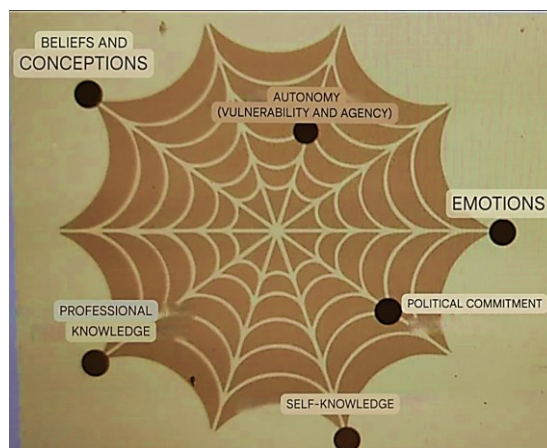
Although they may seem watertight, the macrodomains related to the TTM's PI are deeply interconnected (Cyrino, 2021). Thus, we see those PIs as a *web of relationships* established between these macrodomains: *beliefs/conceptions; self-knowledge; emotions; professional knowledge; autonomy (vulnerability and sense of agency); and political commitment* (Figure 1).

This web represents a system of rhizomatic connections, with no beginning or end, so that by focusing on one of these macrodomains, we inevitably mobilize the others. This simultaneous mobilization can be observed in several situations that correspond to the life and training of teachers: training processes in which they participate; their practices in the classroom; when they look back on their biographical and professional trajectories etc.

According to Thompson (1992, p. 132), “a teacher's conception of the mathematics’

nature can be seen as the conscious or subconscious beliefs of that teacher, the concepts, meanings, rules, mental images and preferences related to the discipline”. Therefore, for Cyrino (2018), beliefs and conceptions emerge from various actions in the professional exercise of mathematics educators, such as the meaning of being a math teacher, the kind of teacher they want to be, how they should teach, their role in the teaching and learning processes, how they should train other teachers, among other actions.

**Figure 1:** Web of relations between the macrodomains that characterize TTM’s PI



**Source:** Research archive

Self-knowledge is related to the process of intersubjectivity and gives meaning to the context in which teachers participate. It results from the interaction’s dynamics regarding what it means to be a teacher and the professional context (Cyrino, 2018). This self-knowledge is based on five components (interconnected and interdependent) proposed by Kelchtermans (2005): self-image, self-esteem, motivations for work, perception of the tasks and future perspectives in relation to the profession.

Alongside self-knowledge, emotions play an important role in the teacher's biopsychosocial constitution. As well as being related to professional well-being, emotions support teachers' work, especially when they experience a scenario of educational changes and reforms. According to Kelchtermans (2005), the emotional impact of these agendas is mediated by the teacher's professional context, related to the dimensions of time (age, generation, biography) and space (structural and cultural working conditions). However, positive and negative emotional experiences, particularly in the context of TTM training, may serve as a springboard for the creation of political micro-actions as a form of resistance and self-transformation (Kelchtermans, 2005).

For Cyrino (2017, p. 702), “the teacher that each person is or will become does not depend simply on the mathematical and didactic knowledge worked in the training processes”. Learning is a personal transformation that starts from the biographical trajectories of the TTM, from their beliefs and conceptions, their various training experiences and their professional performance (or future performance). In this sense, we understand that there is a variety of essential knowledge that deals with the micro-political and contextual realities of the TTM's school life, in order to ensure their professional development.

Autonomy emerges from vulnerability practices and the ways in which the TTM build their professional agency in the socio-professional contexts in which they operate (Oliveira & Cyrino, 2011). Vulnerability is an experience of a multidimensional and multifaceted nature. It is strongly related to the teachers' emotions and involves aspects of different contexts in which they are inserted (Lasky, 2005).

According to Kelthermans (2005), the experience of vulnerability is mediated from the context experienced by the teachers (political environment, social and cultural atmosphere at school, etc.). So, it's directly linked to their PI, but it's not necessarily an emotion. It is a structural condition in which teachers are inserted. On the other hand, the author asserts that “vulnerability is not only a condition to be supported, but also recognized, valued and accepted” (Kelthermans, 2005, p. 999). For this reason, it should not be seen a synonymous with fragility, frustration and helplessness (De Paula & Cyrino, 2021). These conditions seem to be quite contrary to the sense of being, seeing and recognizing oneself as TTM, since, culturally, these subjects tend to put themselves in the condition of exercising a practice free of *mistakes*, *weaknesses* and emotional risks.

The practice of professional agency arises at times when the (future) teacher feels vulnerable. Because he makes choices, makes decisions and reveals his professional and ethical commitment through ideas, motivations, interests and goals (Cyrino, 2017). Cultural resources (values, beliefs, etc.), materials (resources and physical environment) and the social structure itself - in this case, issues involving the forms of relationship, trust, power and roles played - also influence the educator's agency. Moreover, the ability to act socially autonomously and operate independently of the limitations of the social structure (Lopes, Ribeiro, Pazuch & Augusto, 2022) in order to expand their professional knowledge and strive to contribute to the cognitive, social and scientific development of their students, as well as to the professional development of their peers, is also at the basis of the constitution of a teacher's professional agency and, particularly, as TTM.

Cyrino (2017) points out that vulnerability, the search for a sense of agency and political commitment are closely related. She exemplifies that, in many situations, teachers are not in complete control of their working conditions and, therefore, need to be politically committed. For the author, the TTM's political commitment encompasses actions and transformations that consider the commitment to their students, but also to the parents, colleagues and the community in which they are involved.

The following aspects also form the basis of political commitment as TTM: motivations for the work; perception of the roles of mathematics and the work as TTM; prospects for the future and responsibility, i.e. what is the role of the mathematics teacher in the (re)creation of future generations? Who benefits from what the teacher does? (Cyrino, 2017). It is in this context that we will discuss some perspectives on teaching and learning Probability in conjunction with the TTM's PI.

### 3 Teaching and learning Probability from the perspective of the constitution of TTM's PI

The ideas of randomness, chance and risk tend to create a *shifting terrain* for the TTM, considering, among other factors, the incipience of their knowledge to teach Probability; the mobilization of more intuitive and less explicit probabilistic knowledge about the content; and even the skepticism about approaching it in the classroom (Carvalho, 2017). To a certain extent, this seems justified given the educational tradition based on deterministic explanations. Thus, since the nature of Probability is different from the deterministic nature of Mathematics (Batanero & Álvarez-Arroyo, 2023), how can we approach uncertainty in a field that, epistemologically, is based on determinism and on the search for accuracy?

Reflecting on this question bumps into the understanding of the senses and meanings of being and often recognizing oneself as TTM, as the holder of a *monolithic identity*, in this case referring to the privilege of ideas, concepts and deterministic models of Mathematics. On the other hand, the insertion of probabilistic culture within the curriculum of School Mathematics tends to contradict the fact that everything in Mathematics is exact, determined and predictable.

However, running away from it seems to place the TTM in a scenario of contestations of what essentially represents them. Consequently, teachers tend to adapt their view of stochastics and their teaching to problem-solving methods and reasoning patterns used in Mathematics itself.

For Fernandes and Miarka (2015) and Gal (2005), this strategy focuses on technique, i.e. knowledge of mathematical calculations related to Probability, to the detriment of probabilistic thinking, so that students can interpret, reflect and think critically about various probabilistic situations and messages that they may encounter in their daily lives.

Therefore, thinking about the relation between Probability and the socio-political and cultural context is also a demand for its teaching at school and beyond, given that its conceptual, didactic-pedagogical and epistemological bases should provide ways for citizens to deal with uncertainty and risk in the modern world (Gal, 2005; Batanero & Álvarez-Arroyo, 2023). This is a context that has great potential for understanding and promoting some of the questions that hover over the constitution/(re)configuration of the TTM's PI. It considers the ways of being, seeing and recognizing oneself as TTM because, in addition to the knowledge base necessary for teaching (Probability), it involves relations with other aspects, such as beliefs and conceptions, vulnerability and professional agency, political commitment, etc.

For example, when the idea is to teach probability to develop probabilistic literacy (Gal, 2005), many of the aspects (macrodomains) that constitute them as TTM can be activated, reviewed and problematized. According to Gal (2005), being a probabilistically literate citizen means having the cognitive mastery of probabilistic knowledge, as well as dispositions (critical posture, beliefs, attitudes and personal feelings about Probability and risks), linked to the probabilistic messages of the real world. In this sense, reflecting on the social utility of Probability in school education represents a stance based on moral and political commitment as TTM. However, assuming such a stance also tends to put them in a *risk zone*, generating emotional experiences of vulnerability and estrangement, both in the classroom context and in the spaces of continuing education and collaborative work with other TTM.

Thus, teaching Probability from the perspective of Probabilistic Education, i.e. from a perspective that expands the possibilities of working on its teaching and learning, beyond its conceptual and didactic-pedagogical aspect, represents the creation of other ways of acting as a TTM and of building a professional conscience marked by the principles of social justice, professional ethics and equity.

The approach to real and meaningful contexts based on social themes (health, economy, environment, etc.) and the articulated exploration of the various meanings<sup>1</sup> of Probability exemplify the socio-professional awareness that underlies the political commitment of teachers. These are actions that prioritize responsibility as TTM. In this case they are focused on valuing situations of uncertainty and risk in/for the understanding and transformation of subjects, as well as in/for the exercise of critical citizenship.

Although it can be a *shifting terrain* even causing ruptures and destabilizations in the teachers (Nacarato & Grando, 2015), teaching Probability is a potential exercise for the

---

<sup>1</sup> In Batanero (2005), the reader can find a more in-depth discussion of the five probabilistic meanings: intuitive, classical, frequentist, subjective and axiomatic. In this text, it is important to highlight the understanding of intuitive and subjective meanings. The intuitive meaning concerns the first appearances of the use of probability in the modeling of gambling games and how probabilistic vocabulary — such as notions of impossible, most likely, unlikely — was already present in people's daily lives even before formal definitions. The subjective meaning, which in addition to considering the previous and personal experiences of those who declare the probabilities, also takes into account *a priori* and *a posteriori* probabilities and update the quantifications according to new data obtained, can be interpreted as an individual's degree of belief in the occurrence of a certain event (Silva, 2023).

construction of an PI that allows them to develop their self-knowledge (self-image), based on the understanding that being TTM goes beyond the limits of the essentiality of mathematical knowledge.

Not surprisingly, there have been several theoretical and empirical reports on changes in teachers' attitudes and feelings towards Probability and its teaching (Gal, 2005; Estrada & Batanero; 2015; Batanero & Álvarez-Arroyo, 2023). Recently, it has been argued that teachers' aversion and resistance to teaching Probability may be related to their PI, and that this is a pressing agenda for investigation and tension (Gal, 2023).

#### 4 Methodological aspects

The study is based on the qualitative and interpretative paradigm (Alves-Mazzotti, 2001), from a course of Research-Training (Josso, 2007) developed in a study group on Probabilistic Education. Furthermore, in addition to the intention of carrying out a research in a context of teacher training, we seek to perceive the TTM participants as subjects of the training, who were valued personally and professionally in a unique web that involved their psychosocial, cultural and political aspects (Josso, 2007).

We see the act of narrating as a way of understanding the identity experiences of being, seeing and recognizing oneself as TTM (Clandinin & Connelly, 2015). That way, the methodological *design* of the Research-Training sought to create a scenario of narrative (re)construction of personal and professional experiences of the TTM, with the motto of looking at the unveiling of these experiences in the temporal movement that constitutes it and, so peculiar, to the identity phenomenon. Thus, we seek to understand the paths of identity (re)configurations, conceiving the development of PI surrounded by the construction of a Learning Path demarcated by this temporality (Wenger, 1998), in a movement that is dialogic, diachronic and dialectical (De Paula & Cyrino, 2020).

The study group was set up as part of an extension project between the University of Pernambuco (UPE) Petrolina *campus* and the city's public schools. Participating in the group were 11 TTM working in the final years of Elementary School and/or Secondary School (subjects of the research), eight students on the Mathematics Education degree course of the university and two teacher-trainers, the first author of this article and a partner teacher, both lecturers on the course. Twelve face-to-face meetings were held on Saturday mornings at the UPE campus between March and October 2022. Each meeting lasted 3 hours.

Here we focus on two practices of the group and a set of related actions, based on studies on Probabilistic Education. They took place between the 6th and 11th meetings, dealt with the conceptual, epistemological, didactic-pedagogical and curricular aspects of Probability, as well as with the construction of collective lesson plans, their application in schools and the subsequent reflection on the results, also within the teaching group.

Data collection was based on the use of different instruments, namely: audio recording of the meetings and video recording of some of them and subsequent transcription; TTM logbooks and other written productions; reflective diaries referring to the planning and development of classes. The analysis was based on the methodological perspective of the Meaning Cores (MC), conceived as a dialectical process of grasping the meanings (senses and meanings) produced in the group (Aguiar, Aranha & Soares, 2021).

Based on what the constitution of the MC recommends, we started with the election of excerpts/excerpts/speeches narrated by the MSPs, individually and/or collectively, who were impregnated with some meaning as *more significant data*. From these readings and the narrative clues highlighted, we organized a wide range of pre-indicators, in other words, the subjects'

meanings substantiated in meaningful signs/words. The agglutination/articulation of pre-indicators outlining a set of Indicators. This process enabled the emergence of a set of ideas with a different explanatory potential regarding the meanings expressed by the TTM.

Then, we move towards the constitution of the MC as an articulation's movement of the indicators. The process of searching for elements to express the totality of the subjects represented the essence of the MC, mainly as a process of theorizing the research findings more robustly. In the end, a set of seven MC was put together, based on the relations between the macrodomains that form the basis of the TTM's PI characterization web (Figure 1), with the aim of understanding how these macrodomains manifested themselves in the different Learning Paths created from studies on Probabilistic Education. However, in this article, we focus on the discussions of four MC derived from 10 indicators, since they primarily reflect the object and purpose of this work.

## 5 Results and discussion

Studies on Probabilistic Education were based on a set of practices and actions, as well as on the recomposition of pedagogical experiences with the teaching and learning of Probability and its epistemological, conceptual, didactic-pedagogical and curricular aspects. Consequently, they included lesson planning, execution and collective reflections from the perspective of teaching Probability and probabilistic risk (Chart 1).

Initially, the idea was for the TTM to (re)construct a narrative of their previous experiences in this field, considering the ways in which they legitimized their experiences in the group's daily life, the experiences they chose, how they restructured their choices, revised concepts, methodologies and how they began to construct other places for politicizing their mathematical and Probability practices.

**Chart 1:** Group practices and actions around studies on Probabilistic Education

Practices	Shares
1. Recomposing pedagogical experiences with the teaching and learning of Probability and its epistemological, conceptual, didactic-pedagogical and curricular aspects.	1.1 Sand Game (6th meeting); 1.2 Narratives of lessons on Probability from Teaching Cases (7th meeting); 1.3 Epistemological, conceptual, didactic-pedagogical and curricular aspects of Probability and its teaching (8th and 9th meeting).
2. Lessons planning, execution and collective reflections from the perspective of teaching Probability and probabilistic risk.	2.1 Lessons planning on probabilistic risk (9th and 10th meeting); 2.2 Execution and monitoring of lessons in schools (Schools); 2.3 Post-lessons reflections in the group (11th meeting).

**Source:** Elaborated by the authors

As strategies, we made use of the construction of Teaching Cases; the reading and discussion of a basic text focusing on probabilistic literacy; the experience of probabilistic situations/experiments; reflections based on a task about the concept of probabilistic risk; the collective planning of lessons about this concept; the experience in the schools where the TTM work and, finally, the collective reflections on the experience of the lessons in the group.

The Sand Game (Action 1.1) was the group's first contact with Probabilistic Education as a training action. With it, we mobilized the TTM's modes of subjectivation and their cognitive



and dispositional relations (Gal, 2005) regarding Probability and its teaching, based on the construction and reconstruction of scenes in a sandbox. We used various objects that were representative and alluded to the school, the math classroom and Probability itself (dice, coins, bedbugs, African wheelks, playing cards, dominoes, Mega-Sena game tokens, etc.). (Figure 2).

**Figure 2:** Scenes built in the Sand Game



Source: Research archive

Action 1.2 sought to recall some (possible) practices involving Probability content that the TTM had already experienced in the classroom. The idea was for them to narrate how they saw themselves teaching Probability and to review significant episodes in relation to their experiences as students in Elementary School and/or initial training in this field.

In Action 1.3, we carried out three probabilistic experiments, with the initial aim of discussing the notion of equiprobable and non-equiprobable events, using the classical meaning of Probability as a motto. Gradually we moved on to discuss the other meanings (intuitive, frequentist and subjective), explore the perspective of probabilistic literacy (Gal, 2005) and reflect on probabilistic risk.

We moved on to Practice 2, seeking to see other nuances and aspects of the TTM's PI in the light of Probabilistic Education, also considering the reality of some TTM's professional school practice. The motto for planning the lessons (Action 2.1) was to create, replicate or adapt tasks related to the topic of *probabilistic risk* in situations that involved double-entry tables and that could explore some socio-scientific context. This theme was defined by the group, which was the subject of important discussions throughout Practice 1. The purpose was for the planning to include a task developed or adapted from some teaching material - or from the *internet* - involving the proposed topic. Divided into two subgroups, the TTM prepared a lesson plan for the 6th and 9th years of Elementary School, as well as for the 2nd year of Secondary School.

Here, we focus on the results and discussions of the 6th grade class, given the richness of the reflections evoked during the planning phase, at the time of its realization and monitoring at school, as well as after the class. The class was taught by TTM 3 and supported by TTM 11, three undergraduates and the Researcher-Trainer 1, who were also responsible for videotaping the lesson (Action 2.2). Regarding the material of the Secondary School subgroup, we dedicate ourselves primarily to some episodes of reflections in the collective and in the reflective diaries of the TTM responsible for the classes.

The post-class reflections (Action 2.3) took place in the meeting following the applications (11th meeting) and represented an important moment to resume crucial points brought up, notably from the narratives of the teachers in charge, the class observers and the

trainers. We began the meeting with the reading of the reflective diary by each of the TTM responsible for the classes and, therefore, we moved on to the collective's reflections and questions, supported by some excerpts from the video recordings of the classes.

### 5.1 (Inter)weaving meanings of teaching and learning Probability and the movement of (re)configuring the TTM 's PI

The articulations of a set of 10 indicators gave rise to Meaning Cores (MC) 1, 2, 3 and 4 (Chart 2). These four MC condense multiple determinations that contributed to the movement of (re)configuring the TTM 's PI.

**Chart 2:** The indicators and the constitution of the Meaning Cores 1 to 4

Indicators	Meaning Cores
Skepticism regarding the teaching of Probability through the bias of the luck and chance constructs	MC 1 - Beliefs, attitudes and personal feelings towards Probability and its teaching: <i>"I do not like to rely on luck."</i>
The re-signification of teaching knowledge to teach Probability	
Teaching Probability is putting yourself in a risk zone	
Curricular impediments to building a Probabilistic Education	MC 2 - Curricular impediments to the construction of Probabilistic Education based on political commitment as TTM: <i>"You have to take contexts that are real and meaningful for students."</i>
The emergence of a socio-political consciousness from Probabilistic Education: the importance of real contexts in the teaching and learning of Probability	
Challenges and possibilities in building a lesson on probabilistic risk	MC 3 - Providing other ways of teaching and learning Probability based on political commitment as TTM: <i>"I wanted to show it through scientific data."</i>
Individual and collective perceptions about the potentialities and limitations of the developed lesson	
The fears and insecurities revealed during the classes	
The knowledge to teach Probability and probabilistic risk (re)constructed in pedagogical action	MC 4 - The repercussions of planning and reflecting on lessons for future practices in Probability Education: <i>"I feel more motivated and excited to research, learn and teach about Probability."</i>
From reflections on the need for changes in the classes to the consequences of the experience for future practices	

**Source:** Research archive

Each of these MC has senses and meanings that are peculiar to the PI (re)configuration movement observed in the context of the study group. To highlight this, we will discuss each of the MC in the light of the theoretical foundations that underlie the characterization of TTM's PI. It is worth noting that the denomination of each of the MC includes an excerpt from the speech of one of the TTM, highlighted in italics as an *emblem* of the main theme it covers.

MC 1 — Beliefs, attitudes and personal feelings towards Probability and its teaching: *"I don't like to rely on luck"*

The TTM showed skepticism and some initial resistance to working with Probability, motivated mainly by personal and/or dispositional issues (Gal, 2005), based on their *beliefs*, *attitudes* and *emotions* in relation to the way they conceive the antithesis of *luck vs. chance*, which is so present in our sociocultural ideology. The initial strategy of using few objects representing Probability in the Sand Game reinforced this finding.

Although their initial images of Probability and its teaching were determined by their TTM conditions, they have come to understand Probability as a part of Mathematics that deals with uncertainty, risk and randomness (in other words, through inductive thinking), discarding epistemological precepts that go back to Probability theory, which is strongly based on gambling games.

Seeing these epistemological differences and putting the teaching of Probability into perspective based on them has also contributed to changes in their attitudes towards this field of knowledge (Estrada & Batanero, 2015). Whether or not to value the teaching of Probability from the perspective of gambling games alluded to the emotional component of the TTM's training, notably under the argument that they don't like to rely on luck.

*[...] it has nothing to do with Probability, you see? I don't like to rely on luck, I think we must dedicate ourselves and do what we can [...] I think gambling...I have nothing against those who play, but I particularly think it neither influences nor contributes* (TTM 10, Pre-Indicator).

Although they recognized the negative influence of games on students' daily lives, they did not disqualify their relevance in school. At the same time, they safeguarded the importance of overcoming the more *traditional* teaching situations, usually based on working with games that involve the throwing of dice and coins, which almost always incur the bias of equiprobability, i.e. the idea that all possible outcomes are equally likely.

There were also strong indications of TTM's beliefs and attitudes, which endorsed the aversion to Probability when compared to other Mathematics contents, under some arguments such as: believing that Probability is a more difficult content; that students have more questions about it than about other contents; that it deals with a variety of situations, which, although the educators try to act on intuition, are always counterintuitive, making them *walk on eggshells* all the time. Such arguments may be associated with *emotions* and *vulnerabilities*, notably demarcated by their experiences as TTM (or the incipience of them), as TTM 4 narrated:

*But it's often the case that the teacher himself isn't literate. I remember that TTM 9 said it in his research, but I did not know it, so I cannot consider myself literate. So how am I going to say, do this literacy with the students, if I'm not sure myself?* (TTM 4, Pre-Indicator).

In the light of what Estrada and Batanero (2015) have said, the feelings of unpreparedness or that they don't like(d) teaching Probability show the TTM's disaffection and negative attitudes towards the content and, therefore, have repercussions on their emotional conditions. This discovery emphasizes the importance of teachers' affection and emotions related to Mathematics (in this case, to Probability) in the PI's constitution (Gal, 2023).

MC 2 — Curricular impediments to the construction of Probabilistic Education based on political commitment as TTM: *"You have to take contexts that are real and meaningful for students"*

This MC evokes a dilemmatic condition experienced by the group, but one that reflects important aspects of their PIs and of the way they were (trans)formed. Even though they saw

more recent advances in official curricular prescriptions and in textbooks regarding the approach to Probability (exploration of more realistic contexts, extrapolation of situations involving data and coins, etc.), the TTM recounted some curricular impediments that slipped into experiences of *formal/political vulnerability* (Kelthermans, 2005), i.e. that came from factors external to them and consequently tend to trigger *negative emotions*.

According to the TTM, the experimentation with curriculum practice and management in their schools is generally based on a policy of overvaluing external assessments, which, to a certain extent, prevents them from planning Probability lessons that are more autonomous and grounded on the socio-cultural reality of the students.

*I don't know how it works for you, but in the school where I work, Mathematics classes are very focused on external assessments. So, when I think about working Probability it's already for such an exercise, "come on guys, I'm going to teach you how to solve a Probability activity". I download a list of Probability questions with the alternatives and let's go. But something to prepare for an exam, not something for life itself* (TTM 1, Pre-Indicator).

Despite this context, TTM expressed their motivations to work as a mark of *political commitment*. In this sense, they have stressed and problematized the working conditions in their schools and how these values and norms (educational policies based on constant reforms and external evaluations) have affected them personally and professionally (Cyrino, 2017).

Thanks to a whole scenario of conceptual evolution and didactic-methodological, epistemological and ontological ruptures about Probability and its teaching that they demonstrated throughout Practice 1, they also assumed other discourses, suggestive of a transformation of their initial perspectives of Probability as a non-neutral concept, from the development of a critical competence about their roles in the socio-political contexts in which their students are immersed.

*Mathematics isn't just for counting, it's also for you to think about life. For example, when you talk about the Mega-Sena, the chance of winning the Mega-Sena* (TTM 2, Pre-Indicator).

TTM 2, for example, pointed out the socio-political importance of probabilistic knowledge in school and outside of it, and noted that even the contexts of exploiting gambling games are fruitful for developing students' critical sense and empowerment. In this case, related to decision-making in situations of uncertainty and risk in the real world.

In fact, from the moment they pointed out the relevance of other contexts for approaching probabilistic knowledge, as TTM 10 mentioned, *"you have to take contexts that are real and that are meaningful to the students"*, they demarcated their positions and political responsibilities in promoting Mathematics and, particularly, Probability as necessary objects for understanding and transforming the world, whose knowledge generates quality of life and dignity for people.

MC 3 — Providing other ways of teaching and learning Probability based on political commitment as TTM: *"I wanted to show it through scientific data"*

The MC 3 materializes findings that went through the entire process of constructing the lesson plans, as well as the development/execution of these lessons and the reflections on their results, which corroborate new movements in the (re)configuration of the TTM's PI. Notably, we understand that the unveiling of the sense of *professional agency* and *political commitment* emerged more centrally, but together they triggered other aspects that characterize PI, based on

the assumption of individual and collective practices and attitudes.

The specific context in which Practice 2 was developed gave rise to signs of *professional agency*, both individually and mediated by collectivity and collaboration in the group (Oliveira & Cyrino, 2011), even in a curriculum policy scenario which, as has been scrutinized since Practice 1, imposed restrictions and limitations on the TTM's autonomous production capacities.

Thus, building a lesson on probabilistic risk for the 6th grade, even though it is not an object of knowledge recommended by the official documents for this school year, represented an epistemopolitical break for the TTM 3 subgroup and went against the limitations of the social structure, in favor of the cognitive, social and scientific development of the students (Lopes; Ribeiro; Pazuch & Augusto, 2022). Therefore, according to Cyrino (2017), the collective and collaborative agency was guided by choices and decisions that revealed the professional, ethical and political commitment of the group, based on the ideas that emerged, the motivation, interests and objectives in building the lesson plans together.

As TTM 11 recounted, not even the initial uncertainty about adapting the data from the *internet* search on the topic of oral hygiene to the context of a double-entry table was able to deter TTM 3 and the rest of her colleagues from their efforts to create the task. In the post-class reflections, TTM 3's proactivity in carrying forward the construction of the lesson became even clearer, as it dealt with a topic that was already debated in her 6th grade class by the Science teacher.

*I was very happy during the lesson because the topic is one that we're trying to work on a lot with the students. The topic is the issue of oral health, because many of them didn't even brush their teeth before [...]. Then the Science teacher had already started this, he even took the kids to the lab, taught them how to brush their teeth and then I wanted to show this importance with scientific data, complementing the Science lesson, the reason, which is not something random, you brush your teeth because you have bad breath, no, there are consequences and look how many consequences there are and the topic came into my head quickly* (TTM 3, Pre-Indicator).

By saying that she wanted to show the importance of good oral health with scientific data, TTM 3 revealed her *political commitment* emphasizing her perception of the role of knowledge of Probability and risk for the decision-making process and the exercise of critical citizenship by her students (Gal, 2005). In fact, TTM 3 also mobilized *professional knowledge* which, in addition to addressing didactic-pedagogical and curricular aspects, included students' knowledge and the educational contexts in which they are immersed, as well as the aims, values and purposes of the believed education (Shulman, 1987).

Linked to this, it was clear that valuing the interdisciplinary practice that guided TTM 3's lesson and her entire professional attitude was a highlight of the whole experience, not only for her, but also for those who followed the lesson and for the other TTM, especially TTM 7, since he pointed out that:

*[...] I think the lesson wasn't only for her students, at least it was for me too, for us. A very different class, very dynamic* (TTM 7, Pre-Indicator).

On the other hand, several challenges were encountered in the process of planning and developing lessons expressed by the TTM, especially TTM 3. Such challenges evoked experiences of *vulnerability* which, in synergy with their *emotions* (Lasky, 2005), triggered movements to (re)configure their PIs. Positive *feelings* and *emotions* (satisfaction, happiness

and contentment) were evidenced, but also *negative emotions* (fear, insecurity and frustration), expressed in several moments during the planning process, in classes and in the reflection process. In contrast, the exposure of these vulnerabilities also contributed to sustaining individual and collective/collaborative *professional agency* (Oliveira & Cyrino, 2022), since, as a group, they were able to resignify these weaknesses and build other professional awarenesses, with Probabilistic Education studies as their foundation.

MC 4 — The repercussions of planning and reflecting on lessons for future practices in Probability Education: *"I feel more motivated and excited to research, learn and teach about Probability"*

MC 4 reinforces the elements that trigger movements in the (re)configuration of the TTM's PI, primarily from the glimpses of their future professional practices around Probabilistic Education. It highlights the construction of senses and meanings of being, seeing and recognizing oneself as a TTM, which was made possible by the process of observing and monitoring classes, from the perspective of the professional practice on the school floor and the group's collective reflections.

It is noteworthy the aspect of the TTM's *professional knowledge* that was re-signified and/or (re)constructed because of this experience, especially the mathematical knowledge for teaching Probability (content knowledge and didactic-pedagogical knowledge), as TTM 3 said.

*Students had never seen the concept of Probability before. When it came to contextualizing the question "What is the Probability of choosing an adult who doesn't floss?", I changed it to "What is the chance that in a room of 780 people you will make friends with an adult who doesn't floss?"* (TTM 3, Pre-Indicator, Reflective diary).

The concepts of chance and Probability, although they had not been discussed when the collective plan was drawn up, were evoked during the lesson, demonstrating TTM 3's professional *expertise*, especially because it was a 6th grade class. For her, the idea of planning a class in advance from the perspective of collaboration and, above all, based on the idea of always starting with a process of contextualization, was new knowledge. According to her, this went against all her previous planning rituals, which had always been based on explaining conceptual content and practicing exercises from the textbook. Furthermore, by understanding that the focus is always on the student, she recognized that *"bringing the problem and mediating and making them build critical thinking"* (TTM 3, Pre-Indicator) should always be the motto of pedagogical work.

In this context, TTM 11 also said that she began to rethink the teaching of Probability after the experience of planning and observing TTM 3's lesson. Their reflections are in line with what was said by TTM 4 and TTM 8, in the sense that *"approaching Probability in a contextualized way, in order to instigate the students to have a more critical view of situations in their own context and daily life"* (TTM 11, Reflective diary).

The motivations and enthusiasm to research, learn and teach Probability resonated in the collective as potential future practices, from all the discussions that permeated Practices 1 and 2 and the related actions. When TTM 6 felt uncomfortable teaching Probability in one of the lessons planned by her subgroup, she confessed that she had managed to *"detach herself from the direct and 'dry' application of the Probability calculation formula"* (TTM 6, Pre-Indicator) as an ongoing practice.

Likewise, TTM 8 described his new intentions and objectives when teaching Probability, emphasizing the importance of exploring this content from social themes:

*What I'm imagining from that point on, is to try to simulate a survey in the classroom again with social themes and not only explore the question of types of graphs, frequency distribution, but also bring in Probability, you know? Then systematize it (TTM 8, Pre-Indicator).*

We understand, as Oliveira and Cyrino (2022) indicate, that (future) *agency* was fostered/mediated by the TTM's engagement in the group. Thus, feelings of apathy and resistance gave way to motivation and recognition of Probability as one of their future professional tasks (Kelchtermans, 2005). Therefore, the potential developments in the future practices of the TTM were attributed to the collective/collaborative work of the group. They allowed to expand the teaching knowledge base to think and plan lessons on Probability and probabilistic risk from a socio-political and critical perspective. All this was enhanced by the reflective and investigative process developed, both individually and collectively, considering the involvement between peers, the established inter-relational processes of identification (emotions and vulnerabilities), the empathy and solidarity that involved them.

## **6 Final Considerations — Main aspects of the group's practices and actions that indicated the movement of (re)configuration of the PI**

By constructing a globalized analysis between the four MC (internucleus analysis), we can see that the movement of (re)configuration of the TTM's PI was based on the Learning Paths (re)constituted in the collective/collaborative context of the group, inter(woven) by elements of a personal, emotional, moral and socio-political nature that constitute them. In particular, the practices, actions and discourses surrounding Probabilistic Education generated important debates that guided the prospection of other ways of being, seeing and recognizing oneself as such, given the richness of the instruments and resources used in the Research-Training process.

These Learning Paths revealed the (re)construction of personal and professional awareness from the retrospective and prospective movement of looking at themselves (self-knowledge) and the experiences of their peers, undoing limiting beliefs and building strengthening beliefs. The experiences of vulnerability and the aspects of their evoked emotions were mediated both by the socio-professional and cultural context inherent to the TTM and by the training practices instituted in the group. Emotional affinity (positive and negative) was built in the exercise of talking about themselves, exposing anxieties, frustrations and professional limitations (feeling trapped by curricular impositions, for example). This was mainly due to the need to deal with situations inherent to the probabilistic field, which are often conflicting and contingent and *hurt* the constitutive principles of Mathematics as an *exact science*.

As the investigation into teaching and learning Probability deepened, it became clear that political commitment and professional action were important macrodomains that characterized the group's PI, especially after TTM 3's class experience. The construction of lesson plans that went against the usual work based on the context of gambling games in the Probability teaching led them away from the simplistic conception of thinking of games and experiments with dice and coins as the only way to explore this content. Thus, they evoked new conceptions, notably in relation to the perception of the importance of exploring the socio-political contexts that involve decision-making in situations permeated by uncertainty, randomness and probabilistic risk.

Unlike what happens with training processes based solely on *training* teachers, the (re)construction of the TTM's professional knowledge took place in a problematized way, from the moment they reflected conceptually and epistemologically on Probability and its teaching,

which allowed them to see new nuances in the exercise of teaching, on the school floor, moreover, as future practices.

Finally, this work reinforces the need to build more training processes with TTM based on their micro-political realities of training and professional performance, that value horizontal and collaborative relations, with a focus on sensitive listening and empathy. As a training perspective, as characterized in this work, reveals itself as an important investigative niche in the constitution of these training processes, particularly in conjunction with studies in Probabilistic Education.

## References

- Aguiar, W. M. J.; Aranha, E. M. G. & Soares, J. R. (2021). Núcleos de significação: análise dialética das significações produzidas em grupo. *Cadernos de Pesquisa*, 51, 1-16.
- Alves-Mazotti, A. J. (2001). Relevância e aplicabilidade da pesquisa em educação. *Cadernos de Pesquisa*, 113, 39-50.
- Batanero, C. & Alvarez-Arroyo, R. (2023). Teaching and learning of probability. *ZDM Mathematics Education*, 56, 5-17.
- Batanero, C. (2005). Significados de la probabilidad en la educación secundaria. *Revista Latinoamericana de Investigación en Matemática Educativa*, 8(3), 247-263.
- Carvalho, J. I. F. (2017). *Um estudo sobre os conhecimentos didáticos-matemáticos de probabilidade com professores de Matemática dos Anos Finais do Ensino Fundamental*. 344f. Tese (Doutorado em Educação Matemática). Universidade Anhanguera de São Paulo. São Paulo, SP.
- Clandinin, D. J. & Connelly, F. M. (2015). *Pesquisa Narrativa: Experiência e História em Pesquisa Qualitativa* (2. ed.). Uberlândia, MG: EdUFU.
- Cyrino, M. C. C. T. (2017). Identidade profissional de (futuros) professores que ensinam Matemática. *Perspectivas da Educação Matemática*, 10, 699-712.
- Cyrino, M. C. C. T. (2018). Grupos de estudo e pesquisa e o movimento de constituição da Identidade profissional de professores que ensinam matemática e de investigadores. *Revista de Ensino de Ciências e Matemática*, 9, 1-17.
- Cyrino, M. C. C. T. (2021). Ações de Formação de Professores de Matemática e o Movimento de Construção de sua Identidade Profissional. *Perspectivas da Educação Matemática*, 14(35), 1-26.
- De Paula, E. F. & Cyrino, M. C. C. T. (2020). Aspectos a serem considerados em investigações a respeito do movimento de constituição da Identidade Profissional de professores que ensinam matemática. *Educação*, 45, 1-29.
- Estrada, A. & Batanero, C. (2015). Construcción de una escala de actitudes hacia la Probabilidad y su enseñanza para profesores. In: C. Fernández; M. Molina & N. Planas (Ed.). *Investigación en Educación Matemática XIX* (pp. 239-247). Alicante: SEIEM.
- Fernandes, F. S. & Miarka, R. Educar pela probabilidade: um ensaio sobre possíveis educabilidades junto aos pensamentos de Nietzsche e Deleuze. *Quadrante*, 24(1), 81-102.
- Gal, I. (2005). Towards “probability literacy” for all citizens: building blocks and instructional dilemmas. In: G. A. Jones (Ed.). *Exploring probability in school: Challenges for teaching and learning*. (pp. 43-71). New York: Springer.



- Gal, I. (2023, 18 de agosto). *Desenvolvendo letramento estatístico: Direções de ensino e pesquisa*. [Live no Youtube]. [Mediação e Tradução Leandro Souza]. Brasil. Vídeo. Disponível: <https://www.youtube.com/watch?v=tCVDCatWGzo>.
- Graven, M. & Heyd-Metzuyanin, E. (2019). Mathematics identity research: the state of the art and future directions. *ZDM Mathematics Education*, 51, 361-377.
- Josso, M. C. (2007). A transformação de si a partir da narração de histórias de vida. *Educação*, 3(63), 413-438.
- Kelchtermans, G. (2005). Teachers' emotions in educational reforms: Self-understanding, vulnerable commitment and micropolitical literacy. *Teaching and Teacher Education*, 21(8), 995-1006.
- Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education*, 21, 899-916.
- Levy, L. F.; Manfredo, E. C. G. & Gonçalves, T. O. (2012). Concepções sobre Identidade do Professor de Matemática: Portugal e Países Francófonos. *Unión*, 31, 65-74.
- Lopes, C. E.; Ribeiro, R. M.; Pazuch, V. & Augusto, A. F. C. (2022). A coparticipação em um projeto interdisciplinar e a agência profissional docente: narrativas da professora Adriana. *Revista Eletrônica de Educação Matemática*, 17, 1-21.
- Nacarato, A. M. & Grando, R. C. (2015). A análise de aulas videogravadas como prática de formação de professores que ensinam Matemática. In: A. B. Powel (Ed.). *Métodos de pesquisa em Educação Matemática usando escrita, vídeo e internet* (pp. 61-94). Campinas, SP: Mercado de Letras.
- Oliveira, H. M. & Cyrino, M. C. C. T. (2011). A formação inicial de professores de Matemática em Portugal e no Brasil: narrativas de vulnerabilidade e agência. *Interacções*, 7, 104-130.
- Oliveira, L. M. & Cyrino, M. C. C. T. (2022). Aspectos da prática de uma comunidade de professoras de matemática e o desenvolvimento da agência profissional. *Quadrante*, 31(1), 7-27.
- Shulman, L. (1987). Knowledge an teaching: foundations of the new reform. *Harvard Educational Review*, 51(1), 1-22.
- Silva, A. R. S. (2023). *Probabilidade subjetiva no ensino médio: constituição de indicadores epistêmicos e o conhecimento dos estudantes*. 117f. Dissertação (Mestrado em Educação Matemática e Tecnológica). Universidade Federal de Pernambuco. Recife, PE.
- Skott, J. (2018). Re-centring the individual in participatory accounts of professional identity. In: G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt & B. Xu. (Ed.). *Invited lectures from the 13th International Congress on Mathematical Education* (pp. 601-618). Cham: Springer.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In: D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.