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Contributions of the Anthropological Theory of Didactics in the presentation of tasks addressing random events dependent on the paradidactic book "Do I have a chance?"

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Abstract: The objective was to present the theoretical foundation used to create probabilistic tasks that will make up the paradidactic book "Do I have a chance?" for the development of probabilistic concepts in the ninth year of elementary school (approximately 14 years), specifically the notion of dependent random event proposed in the National Common Curricular Base - BNCC. The principles of the Anthropological Theory of Didactic - ATD were used in the didactic and mathematical praxeological organization (probability), which consisted of a sequence of tasks and subtasks, in which techniques were presented and these were justified by the technology that is based on the theory of probability as an object of study. The possibility of developing pedagogical work that involves probabilistic content through everyday playful activities was demonstrated in order to attract the attention of children and young people and generate spontaneous participation and sharing of knowledge.

Keywords: Dependent Random Events. Tasks. Paradidactic Book. Elementary School. Anthropological Theory of the Didactic.

Aportes de la Teoría Antropológica de lo Didáctico en la presentación de tareas que abordan eventos aleatorios dependientes del libro paradidáctico "¿Tengo alguna posibilidad?"

Resumen: El objetivo fue presentar los fundamentos teóricos con los que se crearon las tareas probabilísticas que conformarán el libro paradidáctico "¿Tengo alguna posibilidad?" para el desarrollo de conceptos probabilísticos en el noveno año de la enseñanza primaria (aproximadamente 14 años), específicamente la noción de evento aleatorio dependiente propuesta en la Base Curricular Común Nacional - BNCC. Se utilizaron los principios de la Teoría Antropológica de lo Didáctico - ATD en la organización didáctica y matemática praxeológica (probabilidad), la cual consistió en una secuencia de tareas y subtareas, en las cuales se presentaron técnicas y estas fueron justificadas por la tecnología que se basa en la Teoría de la probabilidad como objeto de estudio. Se demostró la posibilidad de desarrollar un trabajo pedagógico que involucre contenidos probabilísticos a través de actividades lúdicas cotidianas con el fin de atraer la atención de niños y jóvenes y generar participación espontánea e intercambio de conocimientos.

Palabras clave: Eventos Aleatorios Dependientes. Tareas. Libro Paradidáctico. Enseñanza Primaria. Teoría Antropológica de lo Didáctico.

Contribuições da Teoria Antropológica do Didático na apresentação de tarefas que abordam eventos aleatórios dependentes do livro paradidático "Será que tenho chance?"

Resumo: Este artigo tem como objetivo apresentar a fundamentação teórica utilizada para a criação de tarefas probabilísticas que comporão o livro paradidático "Será que tenho chance?" destinado ao desenvolvimento de conceitos probabilísticos no 9º ano do Ensino Fundamental (alunos com aproximadamente 14 anos). Especificamente, aborda-se a noção de evento aleatório dependente, conforme proposta na Base Nacional Comum Curricular (BNCC). Utilizou-se os princípios da Teoria Antropológica do Didático (ATD) na organização praxeológica didática e matemática (probabilidade), resultando em uma sequência de tarefas e subtarefas. Nessa abordagem, foram apresenATDas técnicas justificadas pela tecnologia que se apoia na teoria da probabilidade como objeto de estudo. Mostrou-se a possibilidade de desenvolver um trabalho pedagógico que envolva conteúdos probabilísticos por meio de atividades lúdicas cotidianas, com o intuito de atrair a atenção do público infantojuvenil e promover a participação espontânea e o compartilhamento dos conhecimentos.

Palavras-chave: Eventos Aleatórios Dependentes. Tarefas. Livro Paradidático. Ensino Fundamental. Teoria Antropológica do Didático.

1 Introduction

Teaching probability is important because it helps students develop their probabilistic thinking and reasoning skills, which are required of them as citizens. It also helps them understand different concepts of probability and their applications in different fields of knowledge.

One of these concepts concerns events, often classified as dependent or independent. As a basic rule, the existence or absence of an event can provide clues about other events. In general, an event is considered dependent when it provides information related to another, while it is classified as independent when it does not influence other events.

For events to be considered dependent, the focus of this study, they must have an influence on the probability of another. In other words, a dependent event can only occur if another event occurs first.

Starting from the importance of determining the difference between dependent and independent events, Bennett (2003) states that there is a real need for individuals to have mastery of basic knowledge of probability, believing that this has a relevant role in the formation of citizens. Probability learning contributes to the development of critical thinking, enabling people to understand and communicate a variety of information in diverse situations in everyday life, in which random phenomena, chance and uncertainty are present.

In addition to the relevance of teaching probability, it is noteworthy, according to Campos and Perin (2021), that paradidactic book have been used in Brazil as a complementary resource to the pedagogical process in different areas of knowledge. With a different purpose than the textbook, the paradidactic book seeks to deepen important concepts, using language that is more attractive to the student. Paradidactic books are based on a story filled with illustrations, with the aim of winning over the reader, captivating them through a plot that includes characters that characterize the age group for which it is intended.

Oliveira Júnior and Lozada (2023) systematically analyzed the literature in Brazil that used or created paradidactic book in teaching probability for the Final Years of Elementary School (11 to 14 years old). The authors found twelve studies, from 2014 to 2021. Within the scope of the investigation, which emphasized the methodology or methodological approach used, the development context, the types of study and areas involved, the main results and conclusions, it is highlighted that research with this approach is still in its infancy. A particular concern was highlighted in the research regarding the student's profile and their interactions with experiential and everyday knowledge.

Starting from studies that created statistical or probabilistic tasks according to the principles of the Anthropological Theory of the Didactic (ATD) by Yves Chevallard, Oliveira Júnior, Souza and Datori Barbosa (2019) presented the theoretical foundation used to create problems in the teaching and learning process of statistical content for the 1st year of Elementary School (6 years old). This occurred within the context of didactic and mathematical praxeological organization (statistics) and Stimulus Equivalence. It is believed that using problem solving as a teaching methodology represents an interesting way of presenting statistical concepts. This allows such concepts to be introduced in an attractive way and also encourages creativity in developing resolution strategies and searching for solutions.

In addition to this study, Oliveira Júnior and Datori Barbosa (2023) demonstrated the possibility of developing pedagogical work for the Initial Years of Brazilian Elementary School (6 to 10 years old), based on pedagogical games. This work provides theoretical-methodological support to rethink strategic methods, resizing them in order to minimize the gap between the daily recreational activities carried out. Cards were created for the game “Probability in Action”, called Questions (?), which consist of tasks based on problem situations, with the aim of favoring the apprehension of content and the development of probabilistic knowledge. This approach has theoretical support from ATD, composed of two blocks: practical and theoretical.

Thus, in this study, we sought to present the process of creating tasks that will make up the paradidactic book “Do I have a chance?”, the result of doctoral research. The aim is to encourage reading, develop the ability to interpret texts and make the study of probability for the 9th year of Elementary School (14 years old) more enjoyable, specifically the notion of dependent random event, proposed in the National Common Curricular Base [BNCC] (Brazil, 2018) and in the Report Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) (Bargagliotti *et al*., 2020), supported by ATD.

2 Dependent random events according to the National Common Curricular Base (BNCC) and the North American GAISE II report

Initially, we consider the objects of knowledge and respective probabilistic skills, the focus of this work, indicated in the thematic unit Probability and Statistics of the BNCC (Brazil, 2018), referring to the 9th year of Brazilian Elementary School (Table 1).

Table 1: Objects of knowledge and probabilistic and/or statistical skills according to the BNCC for the 9th year of Elementary School

Knowledge object	Skills
Probability analysis of random events: dependent and independent events.	Recognize, in random experiments, independent and dependent events and calculate the probability of their occurrence, in both cases.

Source: Brasil (2018, p. 318).

At BNCC, there are guidelines for the 9th year of Elementary School to study independent and dependent events in random experiments, in addition to calculating the probability of occurrence in each case. It must be recognized that independent events are those in which the occurrence of one event does not interfere with the occurrence of another, while dependent events are those in which the occurrence of one event is linked to the occurrence of another.

In the GAISE II report (Bargagliotti *et al*., 2020), among the various aspects related to the teaching of statistics, the role of probabilistic thinking stands out, that is, the structure of concepts must be organized at development levels A, B and C. Students at Level A need to develop informal ideas about how probability is related to statistical reasoning.

These ideas will help them when they later use probability to make inferences informally at Level B and more formally at Level C. It should be noted that these levels are not linked to specific stages of teaching. They can be observed progressively throughout schooling but should not be interpreted as directly related to these stages or the age of the students.

Bargagliotti *et al*. (2020) highlight the aspects to be considered at each of the three levels (A, B and C) for teaching probability, directly associated with teaching statistics. Table 2 presents these characteristics.

Table 2: Conceptual framework for teaching probability according to the North American document GAISE II

Level	Description
A	Students should begin to understand that probability is a measure of the chance that something will happen. It is a measure of the degree of certainty or uncertainty. The probability of events should be viewed as being on a continuum from impossible to certain, with least likely, equally likely, and most likely. Using these probabilistic notions, A Level students can craft their answer to the statistical probing question to include statements such as: If a student from our class is selected at random, is it more likely that they will be a student who prefers Rap, Country or Rock?
B	It should be recognized that although the combined sample is larger, it still may not be representative of the entire population (i.e., all students at your school). In statistics, randomness and probability are incorporated into the sample selection procedure to provide a method that is “fair” and to improve the chances of selecting a representative sample. For example, if the class decides to select what is called a simple random sample of 54 students, then each possible sample of 54 students has the same probability of being selected. This application illustrates one of the roles of probability in statistics. Although students cannot actually employ a random selection procedure when collecting data at Level B, issues related to obtaining representative samples should be discussed.
C	It has students go through descriptive statistics of the entire group or population to incorporate notions of chance and probability to make general inferences and inferential comparisons about two groups. Simulations are used to improve probabilistic reasoning. Students understand sample-to-sample variability.

Source: Bargagliotti *et al.* (2020, pp. 27; 49; 72).

Bargagliotti *et al*. (2020) indicate that probability consists of quantifying randomness, being the basis for making predictions in statistics. From a young age, students can use probability informally to predict how likely or unlikely certain events are. It is still possible to consider informal predictions beyond the scope of the analyzed data. Furthermore, it is important to highlight that probability is an essential tool not only in statistics, but also in Mathematics, although it employs different approaches and reasoning than those used in statistics.

According to Bargagliotti *et al*. (2020, p. 10), two problems and the nature of the solutions will illustrate the difference:

> Problem 1: Suppose a die is “fair”.
>
> Question: If a die is rolled 10 times, how many times will we see an even number on the top face?
>
> Problem 2: You pick a die.
>
> Question: Is this a fair data? In other words, does each face have the same chance of appearing?
>
> Problem 1 is a mathematical probability problem. Problem 2 is a statistics problem that can use the mathematical probability model determined in Problem 1 as a tool to search for a solution.

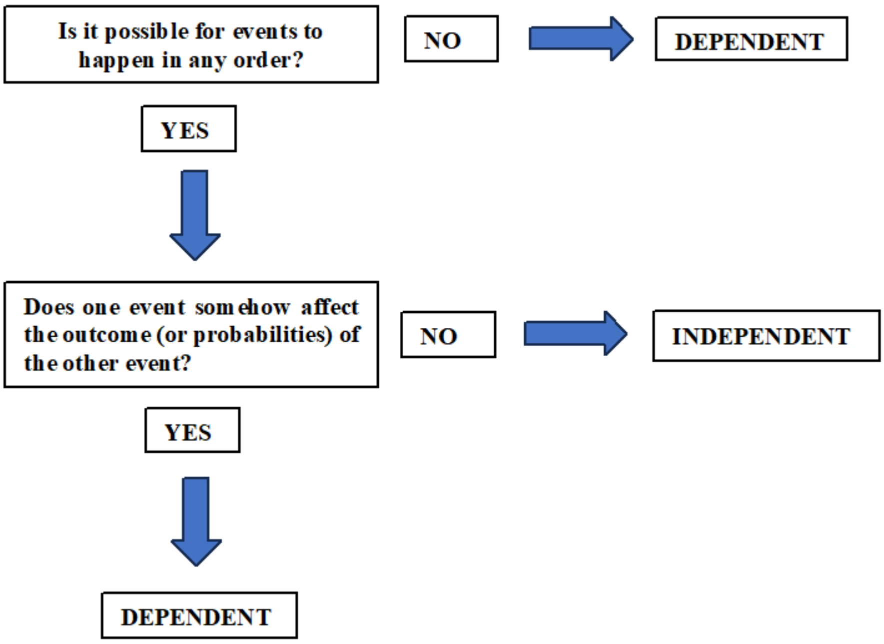
Based on the principles presented at the three levels of the document, a convergence is observed in several directions, mainly with regard to the students' verbalization in relation to their analyses, considering that this action is fundamental for understanding the first probabilistic notions.

In general terms, according to Bargagliotti *et al*. (2020), probability should be understood as a tool used by statistics to evaluate chances of events in contexts guided by research developed by students. However, with regard to the BNCC, this understanding is not clearly evident for all school years of Elementary School (6 to 14 years old). In other words, probabilistic skills do not indicate a connection with a statistical investigative process, which should allow understanding the role of randomness in the analysis and interpretation of data.

3 Identifying dependent random events

For Glen (2023), identifying whether events are dependent or independent can represent a challenge, as not all situations are as simple as they seem at first glance. For example, it can be assumed that your vote for president increases your candidate's chances of winning, but when considering the size of the electoral college, this relationship is not always confirmed. The scheme presented below (Figure 1) indicates aspects that can help in identifying dependent and independent events.

Figure 1: Schema to assist in identifying dependent and independent events



Source: Prepared by the authors. Adapted from Glen (2023).

To complement the understanding of the scheme presented in Figure 1, Glen (2023) suggests identifying steps or stages to differentiate dependent from independent events:

> Step 1 – Ask yourself: is it possible for events to happen in any order? If not (the steps must be performed in a certain order), go to Step 3a. If yes (the steps can be done in any order), go to Step 2. If you are not sure, go to Step 2.
> Step 2 – Ask yourself: Does one event somehow affect the outcome (or probabilities) of the other event? If yes, go to Step 3a, if not, go to Step 3b.
> Step 3a: Done – the event is dependent.
> Step 3b: Done – the event is independent.

4 Methodological procedures

The objective of this research is to present the theoretical foundation used to create probabilistic tasks that will make up the paradidactic book "Do I have a chance?" for the development of probabilistic concepts in the 9th year of Elementary School, specifically the notion of dependent random event, proposed at BNCC (Brazil, 2018) and supported by ATD.

Considering the indicated objective, the following research question was established: how does the process of preparing fictional narrative paradidactic book for teaching probability provide didactic material to facilitate the apprehension of probabilistic concepts in the activities suggested to the student, with the aim of to stimulate your learning, and the teacher, elements that support teaching?

Furthermore, the notion attributed to the task reflects the anthropological meaning of ATD, including only actions that are of human origin – excluding those originating from nature (Chevallard, 1999). It should also be noted that ATD focuses on study activities and is not a teaching or learning theory. Chevallard (1996) uses the term task in ATD and in this text we use it to represent a probabilistic problem.

Bosch and Chevallard (1999) restrict the notion of task in Mathematics by distinguishing mathematical activity from other human activities. When faced with a task, you need to know how to solve it. The "How to solve the task" is the driving force behind a praxeology, that is, it is necessary to have (or build) a technique, which must be based on a technology, which, in turn, requires a foundation on a theory. The word technique will be used as a structured and methodical, sometimes algorithmic, process, which constitutes a very particular case of technique.

Thus, the procedure for preparing the tasks was based on ATD and based on the analysis and discussion of official documents, such as the BNCC (Brazil, 2018), which brings the contents and skills to be worked on in the thematic unit "Probability and Statistics" for the 9th year of Elementary School, and GAISE II (Bargagliotti *et al*., 2020), considering that probability is a measure of the chance of something happening. It is a measure of the degree of certainty or uncertainty. The probability of events should be seen as lying on a continuum from impossible to certain, with least likely, equally likely and most likely lying in between. Using these probabilistic notions, students can develop their answers to a statistical investigative question.

According to Chevallard (1999), to specify a praxeology, it is necessary to understand some fundamental concepts: type of task, task, technique, technology and theory. Thus, according to Bittar (2017, p. 267), the praxeological model proposed to describe any activity, mathematical or not, is composed of:

> type of tasks T; techniques (τ) that solve tasks of this type; technology (θ) that justify the techniques and guarantee their validity, and, finally, the theory (Θ) that justifies the technology. This praxeological quartet is denoted [T, τ, θ, Θ], with the block [T, τ] being called practical-technical (praxis), or know-how block; and the block [θ, Θ] is called technological-theoretical block (logos) or block of knowledge.

In studies conducted by Chevallard (1998) and Chevallard, Bosch and Gascón (2001), it is indicated that a Mathematical Organization (MO) is elaborated around a notion or concept inherent to Mathematics itself or, in the case of this study, probability. They are answers to questions about how to study a particular subject. It refers to the set of tasks, techniques, technologies and theories mobilized to investigate a specific topic (such as probability in the 9th year of Brazilian Elementary School).

In the block considered practical-technical (praxis), the techniques associated with solving a given task will be presented. According to Chevallard (1999), a praxeology related to task T requires, in principle, a way of carrying it out, that is, a way of executing a given task.

In most cases, a certain type of task (T) and its corresponding subtasks (t) are expressed by a verb and its object (Chevallard, 1999). Diogo, Osório and Silva (2007) highlight that, although the concepts of task type (T) and subtasks (t) are closely related, they present differences. The task type (T) can be considered a class of tasks that encompasses several tasks with common characteristics, for example, Task (T) add integers, composed of the following subtasks (t): a) subtask 1: **add** 1 + 2; b) subtask 2: **add** 40 + 50; c) subtask 3: **add** 4 + 2 + 6.

In the block of knowledge (logos), the first component is a rational discourse, called technology (θ) and theory (Θ), which represents a higher level of justification, explanation and production that performs in relation to technology (θ), the same role that this has in relation to the technique -τ (Chevallard, 1999).

Chevallard (1999) proposals for evaluating tasks, techniques, technologies and theories will be taken as a reference. In this way, the tasks aim to be well identified according to the content and reason for their proposal, and whether they are suitable for students in the cycle for which they are intended (9th year of Elementary School). Furthermore, it will be evaluated whether the set of tasks provides a view of the mathematical (probabilistic) situations used in the paradidactic book. The technique will be made available in a complete manner, that is, step by step, or just outlined, and the technology/theory block will be expressed throughout the book, with technological justifications.

Thinking about this praxeology, the elaborate problem situations (tasks) that make up activities contained in the paradidactic book are made up of a sequence of subtasks that can be carried out using different techniques, justified by the technology that uses theories related to probability as an object of study. It is believed that, through ATD, it is possible to broaden the perspective on each proposed task, from the strategies to the theoretical discourse about these strategies.

Chevallard (1999) states that, to begin his theorization, three primitive themes are necessary: objects (O), people (P) and institutions (I). As for the object (O), in this study, emphasis is placed on probabilistic content related to the 9th year of Brazilian Elementary School, supported by the BNCC (Brazil, 2018) and the North American document GAISE II (Bargagliotti *et al.*, 2020).

As for people (P), the focus is to determine guidelines for teachers and students in the 9th year of Elementary School in the process of teaching and learning probability in the construction of their own knowledge, acting critically in the research and development of new methodologies in public entities and private individuals.

According to Chevallard (1999), the institution can be a school, a classroom, a course, a family, etc. Each institution is associated with a set of institutional objects based on the institutional relationship. Thus, the objects are the probabilistic contents indicated for the 9th year of Elementary School. In this context, the institutional role is attributed to the official document that guides the teaching of Mathematics in Basic Education in Brazil at a national level, the BNCC, which covers the different moments of student training.

Finally, Didactic Organizations (DO) were also considered, which, according to Chevallard (1999), present the study process based on didactic moments focused on the functionality of this process. To describe a DO, the same author defines six didactic moments that can appear isolated or simultaneously during the execution of the study, namely:

1) Find OM through at least one type of task, orienting from the beginning to institutional (BNCC) and personal (students and teachers) relationships with the mathematical object (probability).

2) Explore the tasks and develop a technique that allows you to accomplish that task. At this point, the teacher must enable the emergence of other techniques, providing conditions for the student to develop a more exhaustive study of this class of problems.

3) Build a technological/theoretical environment that is closely related to previous moments.

4) Work on the technique for different tasks, which may eventually be changed and/or improved.

5) Define MO, highlighting elements that were part of the study, that is, institutionalization

6) Evaluate the institutional and personal relationships produced around the object of study.

5 The teaching of probability in the paradidactic book “Do I have a chance?” for the 9th year of Elementary School in light of ATD: tasks that address the concept of dependent random event

Below are the tasks that make up the paradidactic book “Do I have a chance?” for the 9th year of Elementary School, following the principles of ATD in the didactic and mathematical (probabilistic) praxeological organization.

With regard to mathematical and didactic organization, it occurs at two distinct moments. The first moment is called elaboration and systematization of the techniques chosen to solve the proposed problems, which can be characterized as dependent random events (subtypes of tasks). These techniques are explored in introductory situations and are explained during the resolution process. It is at this moment that the properties or statements that constitute the technological elements that explain or justify the systematized techniques are stated. The second moment, called technical work, occurs through the performance of the tasks presented and then, through the systematization process.

Furthermore, dependent events are considered to be important in both statistics and probability, in the context of everyday life, as they can affect the way probability is calculated and influence decision-making based on statistical analysis. Therefore, it is important to understand the dependency between events and take it into account in data analysis.

5.1 Tasks related to the preparation of the Sete Colinas School team before the national stage of the Probability Olympiad

Considering the paradidactic narrative, it was determined that the group of students, characters created for the story, reached the final stage of the National Probability Olympiad. Back to activities at Escola Sete Colinas, teacher Valéria gathered her students to congratulate them on, yet another stage completed.

After winning another stage (state), the Sete Colinas group returned to their city, and the students were eagerly awaiting the last stage that would take place in a month in the country's capital. Thus, in one of the preparation moments, the teacher Valéria sought to review the content with the group, when one of the team's students, Rafael, asked:

> *Teacher, do we have a chance of winning this Olympics?*
>
> The teacher Valéria said, looking fixedly at everyone in the group:
>
> *Of course, yes! You dedicated yourselves throughout the Olympics and there is no reason to worry now!*
>
> And she continued:
>
> *You have the same chance that other groups have. And taking advantage of your question, what is your probability of becoming national champions?*

The group looked at each other and began discussing what procedures they could follow to determine this probability. After a while, Gabriela, who was nominated by the group to present the ideas they had, said:

> *Well! If we have the same chance as the other groups and considering that in Brazil we have 26 States and the Federal District, then the probability is 1/27, that is, approximately 0.037, and if we multiply this value by 100, we obtain 3.7 %. Thus, the probability of becoming champions is approximately 3.7%.*
>
> Luíza seemed discouraged because the probability was small, but said:
>
> *Don't be discouraged!! The probability we have is the same as the other groups, so let's get excited!*
>
> And teacher Valéria continued:

That's right! This is no time for discouragement. Furthermore, we have to learn the difference between dependent and independent random events! Speaking of which, does anyone remember how to differentiate dependent from independent events?

Thinking about the question that teacher Valéria asked, showing a great desire to learn more, Rafael asked:

Teacher Valéria, a question arose!!! If I want to select a shirt from my closet to wear on Monday and then a different shirt to wear on Tuesday, are these events dependent or independent?

To help teacher Valéria or other teachers who want to use this task in their classes, Task 1 is considered "Recognize, in everyday random experiments, dependent events". Starting from Task 1 (T_1), a sequence of subtasks (t_1 to t_3) are presented according to ATD and its mathematical and didactic praxeological organization.

Thus, considering the situation presented by Rafael, in Table 3, the practical-technical block (praxis) or know-how is described, referring to subtask 1 (t_1) associated with Task 1 (T_1).

Table 3: Description of the practical-technical block (praxis) or know-how, referring to subtask 1 (t_1) associated with Task 1 (T_1), according to ATD

Subtask 1 (t_1)	Consider the events A = {selecting a shirt from the closet to wear on Monday} and B = {selecting a shirt other than the one selected on Monday from the closet to wear on Tuesday} and recognize whether these events are dependent or independent.
Technique ($|_1$)	Two events are considered to be dependent if the outcome of the first event affects the outcome of the second event. Thus, in subtask 1 (t_1), events A and B are dependent, because, after the first shirt is chosen on Monday, there are fewer shirts to choose on Tuesday, since there is no possibility of choosing the first shirt again. Monday shirt, that is, Rafael cannot choose the same shirt twice.

Source: Prepared by the authors (2024).

As the team continues to prepare for the Olympics, Luzia expressed her desire to contribute, bringing up a situation she experiences daily when going to school, and asked:

I take the bus from my house to school every day. Let the event be A I arrive at the bus stop in time to catch the bus and the event B be I arrive at school on time. Are these events dependent or independent?

Thus, considering the second situation, the one presented by Luzia, Table 4 describes the practical-technical block (praxis) or know-how, referring to subtask 2 (t_2), also associated with Task 1 (T_1).

Table 4: Description of the practical-technical block (praxis) or know-how, referring to subtask 2 (t_2) associated with Task 1 (T_1), according to ATD

Subtask 2 (t_2)	Consider the events O = {arrive at the bus stop in time to catch the bus} and E = {arrive at school on time} and recognize whether these events are dependent or independent.
Technique ($|_2$)	Two events are considered to be dependent if the outcome of the first event affects the outcome of the second. Thus, event O is Luzia who arrives at the bus stop in time to catch the bus, and event E is Luzia who arrives at school on time. We can assume that the probability of arriving at school on time increases greatly if the student takes the bus, as he or she is more likely to be late for school if he or she misses the bus. Thus, the occurrence of event O affects the probability of event E occurring, meaning that the two events are dependent.

Source: Prepared by the authors (2024).

Still trying to contribute, Gabriela, aware that her mother is pregnant, thought about the following situation:

> *You know my mother is pregnant. Then I thought about the following situation: A family has 4 children, and we are going to select two of these children at random. Let F_1 be the event in which a child has blue eyes and F_2 be the event in which the other chosen child also has blue eyes. Are these events dependent or independent?*

A third situation or task was presented by Gabriela. Thus, in Table 5, the practical-technical block (praxis) or know-how is described, referring to subtask 3 (t_3) still associated with Task 1 (T_1).

Table 5: Description of the practical-technical block (praxis) or know-how, referring to subtask 3 (t_3) associated with Task 1 (T_1), according to ATD

Subtask 3 (t_3)	Consider the events F_1 = {a child has blue eyes} and F_2 = {another chosen child also has blue eyes} and recognize whether these events are dependent or independent.
Technique (τ_3)	In this case, F_1 and F_2 are dependent and can be justified by the following aspects: 1) It is assumed that eye color is hereditary. Thus, the fact that a child has or does not have blue eyes will increase or decrease the chances that another child in the same family will have blue eyes, respectively. 2) The result of the first event affects the result of the second, because, after randomly selecting and identifying that the first child has blue eyes, there would be fewer children, in addition to also having blue eyes in the second selection. In the selection of the second child, sampling without replacement would also be considered, that is, the result of the selection of the second child depends on the selection of the first child.

Source: Prepared by the authors (2024).

Regarding subtasks 1 to 3 (t1 to t3) associated, respectively, with theories 1 to 3 (τ_1 to τ_3), a technological-theoretical block (logos) or knowledge block is described, that is, technology θ_1 and the theory Θ_1. In other words, these blocks depend on what happened previously. They are influenced by the outcomes that have already happened, i.e. two or more events that depend on each other are known as dependent events. Two events can also be considered to be dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second, such that the probability is changed.

Considering subtasks 1 to 3, Kaori presented the class and teacher Valéria with an activity that she copied from a textbook that she took from the school library (Figure 2).

Figure 2: Image of the activity copied and then typed into a word document by Kaori

In a bag there are two white and four black balls. Two balls are drawn simultaneously.
1) Consider the randomized experiment "removing two black balls from the bag". Indicate whether these events are dependent or independent?
2) What is the probability that both balls are black?

Source: Prepared by the authors (2024).

Task 2 is considered "Recognize, in everyday random experiments, dependent events and calculate the probability of their occurrence". Starting from Task 2 (T_2), a sequence of subtasks (t_4 to t_5) is presented according to ATD and its mathematical and didactic praxeological organization.

Starting from the situation identified by Kaori in a textbook, in Table 6, the practical-technical block (praxis) or know-how is described, referring to subtasks 4 and 5 associated with Task 2 (T_2).

Table 6: Description of the practical-technical block (praxis) or know-how, referring to subtasks 4 and 5 (t_4 to t_5) associated with Task 2 (T_2), according to ATD

Subtask 4 (t_4)	Consider the events P_1 = {remove the first ball from the bag and check that it is black} and P_2 = {remove the second ball from the bag and check that it is black} and recognize whether these events are dependent or independent.
Technique 4 (τ_4)	The two events are considered to be dependent, as the result of the first event affects the result of the second. Thus, events P_1 and P_2 are dependent, because, after removing the first ball from the bag and verifying that it is black, there are fewer black balls in the second selection of another black ball. It is also highlighted that sampling is carried out without replacement, with the result of the second extraction depending on the result of the first.
Subtask 5 (t_5)	Considering the random experiment "removing two balls simultaneously from a bag in which there are two white and four black balls", calculate the probability of two balls being black.
Technique 5 (τ_5)	Since there are two white and four black balls in the bag, the probability of a black ball in the first selection is 4/6 = 2/3. In the second part, the second event depends on the first and the probability of obtaining another black ball is modified: *P* (2nd black ball) = 3/5. That is, in the first selection there were 4 black balls in a total of 6 balls, however, in the second selection, there are only three black balls in a total of 5 balls. Thus, *P* (two black balls) = *P* (1st black ball and 2nd black ball) = *P* (1st black ball ∩ 2nd black ball) = 2/3 × 3/5 = 6/15 = 0.4 or 40%.

Source: Prepared by the authors (2024).

Considering subtask 4 (t_4) associated with technique 4 ($|_4$), a technological-theoretical block (logos) or knowledge block is described in the same way as technology $\llcorner_1$ and theory $\cup_1$, previously described.

In the case of subtask 5 (t_5) associated with technique 5 ($|_5$), technology $\llcorner_2$ can be described according to Anderson, Sweeney and William (2012), while the law of multiplication or product that is used to calculate the probability of the intersection of two events, being based on the definition of conditional probability.

Considering the studies by Glen (2023), two events are considered dependent when the occurrence of one influences the probability of the other. Probability-dependent events reflect common real-life situations. For example: 1) If you want to attend a concert, this may depend on whether you receive overtime at work; 2) If you plan to visit family outside the country next month, this may depend on whether or not you can get a passport in time.

The $\cup_2$ theory, which explains and justifies the $\llcorner_2$ technology and the $|_4$ technique, is initially based, according to Smole and Diniz (2010, p. 12), as follows:

> If an event A_1 can occur in m_1 different ways, for each way A_1 occurs, an event A_2 can occur in m_2 different ways, and for each way A_1 and A_2 occur, an event A_3 can occur in m_3 different ways: then, the number of different ways for A_1, A_2 and A_3 to occur is $m_1 x m_2 x m_3$.

Complementando a lei da multiplicação e produto, para Magalhães e Lima (2005), dados dois eventos A e B, a probabilidade condicional de A dado que ocorreu B pode ser representada por P(A/B) e dada por $P(A/B) = \frac{P(AB)}{P(B)}$, P (B > 0). Assim, considerando as equações (1) e (2),

Complementing the law of multiplication and product, for Magalhães and Lima (2005), given two events A and B, the conditional probability of A given that B occurred can be represented by P(A/B) and given by $P(A/B) = \frac{P(AB)}{P(B)}$, P (B > 0). Thus, considering equations (1) and (2),

$$P(A/B) = \frac{P(AB)}{P(B)} \tag{1}$$

$$P(B/A) = \frac{P(AB)}{P(A)} \tag{2}$$

and calculating P (A) B), we obtain the law of multiplication or product, indicated in (3) and (4):

$$P(AB) = P(B) * P(A/B) \tag{3}$$

$$P(AB) = P(A) * P(B/A) \tag{4}$$

Based on the laws previously presented, it can also be considered that, when two events A and B are dependent, the probability of occurrence of A and B is indicated by expressions (3) or (4).

Finally, Kauê, who had already learned a lot about dependent random events and was curious to know about color blindness, a condition of one of his brothers, presented everyone with research he did on the internet:

It is known that in the general population there are 51% men and 49% women and that the proportions of colorblind men and women (reduced ability to distinguish certain colors) are presented in the following table:

	Mem (B1)	***Woman (B2)***	***Total***
Being colorblind (A1)	*0,040*	*0,002*	***0,042***
Not being colorblind (A2)	*0,470*	*0,488*	***0,958***
Total	***0,510***	***0,490***	***1,000***

Furthermore, if a person is randomly selected from this population and is found to be a man (event B1), what is the probability that the man is colorblind (event A1)?

Task 3 is considered "Calculate the probability of an everyday problem". Starting from this task, the subtask (t6) is presented according to the ATD. In Table 7, the practical-technical block (praxis) or know-how is described, referring to subtask 6 associated with Task 3 (T_3).

Table 7: Description of the practical-technical block (praxis) or know-how, referring to subtask 6 (t_6) associated with Task 3 (T_3), according to ATD

Subtask 6 (t_6)	Considering the random experiment "select a man from the population and he is color blind" and if a person is randomly selected from this population and is considered a man (event B1), calculate the probability of the man being color blind (event A1).
Technique 6 ($|_6$)	The two events are considered to be dependent, as the result of the first affects the result of the second. When we know that event B occurred, attention is restricted to only the 51% or 0.51 of the population that is male. It is also worth noting that, in the table presented, if you identify

	yourself knowing that the person is male, the probability of being color blind is equal to 4% or 0.04. Thus, P (Person being colorblind - event A1/person is a man - event B1) = P(A)*P(B/A) = 0.51*0.04 = 0.0204 or 2.04%.

Source: Prepared by the authors (2024).

The $\cup_3$ theory, which explains and justifies the $\lfloor_3$ technology and the $|_6$ technique, is based on Berenson (2008) and Tavares (2014). When checking whether two events are independent or dependent, it is necessary to define the concept of conditional probability of one event in relation to another and differentiate between inclusive events.

According to Berenson (2008) and Tavares (2014), the conditioned probability of two inclusive events is defined as the probability of event A occurring, whenever event B occurs, that is, $P(A/B) = \frac{P(A \cap B)}{P(B)}\ ou\ P(AB) = P(B) * P(A/B)$. Therefore, the conditioned probability is the probability that A and B occur divided by the probability of B occurring. Furthermore, one can also define the probability that B occurs conditional on A, that is, $(B/A) = \frac{P(A \cap B)}{P(A)} ou\ P(AB) = P(A) * P(B/A)$. In this case, the events are dependent.

5.2 Tasks related to the participation of the Sete Colinas School team in the national stage of the Probability Olympiad

It is noteworthy that, as part of the narrative of the paradidactic book, the idea was to use a game that addressed probabilistic content to consolidate and establish the objective through which the idea of creating a paradidactic book emerged. The teaching objective was to use games of chance, which are those in which loss or gain depend more on luck than on calculation, or just on luck, with these games being closely linked to probabilities.

It was considering bingo, a game of chance in which numbered balls are placed inside a globe and drawn one by one, until a player fills his entire card with these results. Traditionally, the winners are those who manage to fill in the card first, and when they do so, they must announce that they have won by shouting the word “BINGO!”.

The game “Odds Bingo” was also considered, in which cards are provided to the leading players of each group, containing twenty-five representations of 5x5 numbers (five by five), related to the number of letters in the word BINGO.

The game works like traditional bingo, however, the cards contain questions about probability. Each player marks the values on their card if they are the correct answer to the drawn question. This game makes up the last stage of the Olympics, which will take place at a national level, and the group made up of the main characters will have to go through this stage to win the Olympics.

It is worth remembering that participation in the game assumes that the student has studied the probabilistic contents of this stage. When paradidactics are used in the classroom, they can be designed to assess students' prior knowledge, direct teaching or even fix content. In the plot of the paradidactic book's story, during the competition, when the team determines the solution on an official Olympiad sheet of paper, they must mark the corresponding value on their Bingo card.

Therefore, Table 8 presents five problems aimed at probabilistic concepts for the ninth year of elementary school, specifically dependent random events. These problems will appear in printed material made available by the event organizers and, consequently, will start the game and the final stage of the Olympics.

Table 8: Problemas presentes da Olimpíada de Probabilidade referentes a eventos aleatórios dependentes

Competition issue	Statement
1	A six-sided die is rolled. Event A is getting an even number and event B is getting a prime number. Determine P (A ∩ B).
2	Two candies are picked at random from a bag containing eight chewy candies and two crunchy (non-chewable) candies. Event A is picking a chewy candy first and event B is picking a crunchy candy in the second selection. Calculate P (Take a chewy candy in the first selection and Take a crunchy candy in the second selection).
3	Two classmates are randomly selected from a group containing five boys and five girls. Event A is to select a boy first and event B is to select a girl second. Calculate the following probability: P (selecting a boy first and selecting a girl second).
4	We would like to select two students to help with the elaboration of questions about random events from one of the teams participating in the Probability Olympiad, that is, 3 girls and 2 boys. What is the probability that those selected will be two boys?
5	A bookstore located close to the final stage of the Olympics periodically receives new shipments of probability books for elementary school. In fact, the probability of receiving new books on any given day is 76%. If a shipment of new books is expected, there is a 30% probability that books about probabilistic concepts (dependent random events) that you want to purchase will be in that shipment. What is the probability that there will be a shipment of new probability books today and that the book on probabilistic concepts (random events) will be in that shipment?

Source: Prepared by the authors (2024).

Task 4 is considered "Calculate the probability of an everyday problem after recognizing that it is configured as a dependent random experiment". Starting from this task, a sequence of subtasks (t_7 to t_{11}) is presented according to the ATD and its mathematical and didactic praxeological organization. In Table 9, the practical-technical block (praxis) or know-how is described, referring to subtasks 7 and 11 associated with Task 4 (T_4).

Table 9: Description of the practical-technical block (praxis) or know-how, referring to subtasks 6 to 10 (t_6 to t_{10}) associated with Task 3 (T_3), according to ATD

Competition issue	Subtask	Technique
1	Subtask t7 - Considering the random experiment of "rolling a six-sided die twice", calculate the probability of obtaining an even number on the first roll and a prime number on the second.	Technique $|_7$ - Event A is an even number {2,4,6}, while B is a prime number {2,3,5}. The probability of getting a prime number is $P(B)$=1/2 because there are 3 prime numbers out of 6 equally likely outcomes. On the other hand, if we assume that an even number has already been selected, then the result has to be a 2 for it to be a prime number. In other words, the event B given A is the same as getting 2 from {2,4,6}. So, $P(B\|A)$ = 1/3. Since $P(B\|A) \neq P(B)$, the two events are dependent. To indicate the probability associated with this random experiment, we have that: P(A∩B) = P(A)*P(B/A) = 1/2*1/3 = 1/6 = 0.1667 or 16, 67%.

2	Subtask t8 - Considering the random experiment "remove two candies at random from a bag that contains eight chewy candies and two crunchy candies", calculate the probability of the first being an edible candy and the second a crunchy candy.	Technique τ_8 - Event *A* is the child picking up a chewy candy first, and event *B* is the child picking up a crunchy (not chewy) candy second. To determine whether the two events are independent or dependent, it is necessary to identify whether the probability of event *B* occurring is different depending on whether event *A* has already occurred or not. If the chewy candy is selected in the first selection, event *A* is said to have occurred and P (a chewy candy is produced in the first selection) = 8/10 = 2/5. Considering that the candy remains outside the bag (without replacing it in the bag) and making a second selection of what is left in the bag, it is in this selection that a crunchy candy is selected, so $P(B\|A)$ = 2/9. Since $P(B\|A) \neq P(A)$, the two events are dependent. To indicate the probability associated with this random experiment, we have that: P(A∩B) = P(A)*P(B/A) = 2/5*2/9 = 4/45 = 0.0889 or 8, 89%.
3	Subtask t9 - Considering the random experiment "select two children from a peer group of five boys and five girls", calculate the probability of the first being a boy and the second a girl.	Technique τ_9 – Let the event *A* select a boy first and the event *B* select a girl, second in a group with 2 boys and 3 girls. Thus, the probability of selecting a girl second depends on whether a boy was selected first – P(A) = 2/5, since the composition of the remaining group for the second selection will be different. Thus, since event *A* occurred, there will be 2 boys and 2 girls available for the second selection and $P(B\|A)$ =2/4 = 1/2. Since $P(B\|A) \neq P(A)$, the two events are dependent. To indicate the probability associated with this random experiment, we have that: P(A∩B) = P(A)*P(B/A) = 2/5*1/2 = 2/10 = 1/5 = 0,2 or 20%.
4	Subtask t10 - Considering the random experiment "select two students participating in the Probability Olympiad to ask questions from a group of two boys and three girls", calculate the probability of the two being boys.	Technique τ_{10} - Total number of students participating in the group participating in the Probability Olympiad = 3 girls + 2 boys = 5 students. The Probability of choosing the first boy, say P (selecting first boy) = 2/5. P (select second boy/select first boy) = P(B/A) = 1/4. Furthermore, as $P(B\|A) \neq P(A)$, the two events are dependent. Finally, P (select first boy and select second boy) = P (select first boy) * P (select second boy/select first boy) = (A)*P(B/A) = (2/5) × (1/4) = 2/20 = 1/10 = 0.1 or 10%.
5	Subtask t11 - Considering the random experiment "to have a book on probabilistic concepts (random events) in a batch of new probability books",	Technique τ_{11} - Let A = "a shipment of new probability books arrives today" and B = "the book you want to buy is in this shipment". Firstly, it is identified that the two events are dependent, because when sending new probability books, there may be concepts that address dependent random events that interest you. The question also asks to determine the probability that new

	calculate the probability that the book on probabilistic concepts (random events) will be in this batch of new books.	probability books arrive and that the book you want to buy (dependent random events) is among them, that is, you want to determine P (A and B) or P (A) B). The question also indicates that P(B/A) is 30% or 0.30, that is, "as long as the bookstore receives new probability books today, your book will be among them (dependent random events)". Therefore, using the following equation for conditional probability, P(B/A) = (P(A∩B))/(P(A)), we obtain: 0.30 = (P(A∩B))/0.76. So: P(A∩B) = 0.30 x 0.76 = 0.228 or 22.8%

Source: Prepared by the authors (2024).

Theory $\cup_4$, which explains and justifies technology $\llcorner_4$ and techniques $|_7$ to $|_{11}$, complements theory $\cup_3$, which explains and justifies technology $\llcorner_3$ and technique $|_6$, as discussed by Berenson (2008) and Tavares (2014), or differentiates exclusive events from inclusive events.

For Berenson (2008), two events are said to be exclusive if the possible values or elements of one event A have nothing in common with the values or elements of another, called B. Therefore, in two exclusive events, the set of the intersection of A with B is emptiness: A∩B = Ø. On the contrary, if events are inclusive, it may occur that one or more outcomes of event A coincide with those of another event B, with A and B being different events. In this case: A∩B ≠ Ø.

For Berenson (2008), three criteria are considered to know whether two events are independent, as long as one of the three is met, for the independence of the events to be demonstrated. Otherwise, the events will be dependent. Like this:

1) If the probability of A happening whenever B occurs and is equal to the probability of A, then they are independent events, that is, P (A/B) = P (A) => A is independent of B.
2) If the probability of occurrence B given A is equal to the probability of B, then there are independent events, that is, P (B/A) = P (B) => B is independent of A.
3) If the probability that A and B occurs is equal to the product of the probability that A occurs by the probability that B occurs, then these are independent events. The converse is also true. Thus, P (A∩B) = P (A) * P (B) <=> A and B are independent events.

6 Final considerations

Thinking about probabilistic concepts as dependent random events is to develop research that meets the needs of Brazilian basic schools, to contribute to the growth and development of an autonomous, critical, active society and capable of making decisions in light of the information with which it comes across.

It is also believed that, through ATD and Mathematical and Didactic Organization, it is possible to broaden the perspective in relation to the various existing possibilities that surround each task presented in this text, designed from “knowing” to probabilistic “doing”.

The aim is to contribute to the teaching and learning processes of students and teachers at the end of Elementary School. It is conceived that the creation of these tasks, contained in the narrative of children's literature, will provide the student with moments of pleasure, exchange and learning, and the teacher, a theoretical resource to approach the contents that are worked on.

Regarding ATD, its use allowed the identification of a set of praxeologies that make it possible to characterize both the mathematical object (probabilistic) and the didactic approach to this object. The praxeological organization was composed of four elements:

1. Task (T) and its subtasks (t): characterized the action required by the problem situation proposed for the tasks contained in the textbook. For example, calculating the probability of an everyday problem after recognizing that it is configured as a dependent random experiment.
2. Technique (|): identifies how the task and its subtasks are carried out. Each task has at least one technique associated with it. For example, determining whether two events are dependent. Considering other experiments, we will have other combined possibilities such as the events being independent.
3. Technology (\)/Theory (◡): was specified by the set of definitions, properties, axioms and theorems that justify the technique. For example, the technology that justifies the technique is the definition of a dependent random event, that is, for each random experiment E, the sample space S is defined as the set of all possible results of that experiment. It is considered that there are several ways to find the probabilities of dependent events based on the Fundamental Counting Principle.

In addition to the contributions highlighted by ATD, this study seeks to highlight that it is not enough to just have skills related to probabilistic calculation in the teaching and learning process. It is necessary to observe the context in which the data is inserted. In other words, tasks must be designed considering their application in the classroom context. In addition to mastering calculation, it is necessary to be able to explain the behavior of data.

The research could still have been guided, in a similar way, by the probabilistic literacy model and its cognitive elements proposed by Gal (2005), namely: 1) Big Ideas: randomness, independence, variation, predictability and uncertainty and others; 2) Probabilistic Calculations: different ways of finding or estimating the probability of events; 3) Language: the terms and methods used to express probabilistic results; 4) Context: understanding the role and meanings of probabilistic messages in different contexts; 5) Critical questions: reflections on issues in the context of probability. These aspects would be associated with the importance of mobilizing and coordinating records of semiotic representation from the perspective of Duval (2016) for learning probability.

The idea is added that the apprehension of this concept does not occur in a simple way, even because they are not simple, but it is believed that the didactic transposition adopted in everyday classroom life will make all the difference in the teaching and learning process.

It is reinforced according to Bargagliotti *et al*. (2020) that probability deals with quantifying randomness, being fundamental for making statistical predictions. It is important to start developing these concepts from a young age, creating situations in which students can use probability informally. This allows them to consider informal predictions beyond the scope of the data they analyzed.

The creation of the tasks that make up the paradidactic book is justified by the fact that the narrative told in a playful environment attracts the attention of children and young people, who spontaneously participate and share their knowledge.

References

Anderson, D. R.; Sweeney, D. J. & William, T. A. (2012). *Estadística para Negocios y Economía*. (11. ed.). México: Cengage Learning.

Bargagliotti, A.; Franklin, C.; Arnold, P.; Johnson, G.; Rob, S.; Perez, L. & Spangler, D. A. (2020). *Pre-K–12 Guidelines for assessment and instruction in statistics education II* - A framework for statistics and data science education. An official position of the National Council of Teachers of Mathematics as approved by the NCTM Board of Directors.

Bennett, D. J. (2003) *Aleatoriedade*. Trad. de W. Barcellos. São Paulo: Martins Fontes.

Berenson, M. (2008). *Estatísticas para Administração e Economia*. (10. ed.). São Paulo: Pearson Universidades.

Bittar, M. (2017). A Teoria Antropológica do Didático como ferramenta metodológica para análise de livros didáticos. *Zetetiké*, 25(3), 364-387.

Bosch, M. & Chevallard, Y. (1999). Ostensifs et sensibilité aux ostensifs dans l'activité mathématique. *Recherches en Didactique des Mathématiques* (pp. 77-124). Grenoble: La Pensée Sauvage.

Brasil. Ministério da Educação. Secretaria de Educação Básica. (2018). *Base Nacional Comum Curricular:* Educação Infantil e Ensino Fundamental. Brasília, DF.

Campos, C. R. & Perin, A. P. (2021). Livro paradidático: um estudo voltado para o ensino/aprendizagem de Estatística na escola básica. *Educação Matemática Pesquisa*, 23(4), 140-170.

Chevallard, Y. (1996). Conceitos Fundamentais da Didática: as perspectivas trazidas por uma abordagem antropológica. In J. Brun. (Org.). *Didáctica Das Matemáticas*. Tradução de M. J. Figueredo. Instituto Piaget.

Chevallard, Y. (1998). Analyse des pratiques enseignantes et didactique des mathematiques: L'approche anthropologique. *Actes de l'U.E. de la Rochelle*.

Chevallard, Y. (1999). L'analise des pratiques enseignantes en théorie antropologique du didactique. *Recherches en Didactique des Mathématiques. La Pensée Sauvage-Editions*, *19*(2), 221-265.

Chevallard, Y. ; Bosch, M. & Gascón, J. (2001). *Estudar matemáticas*: o elo perdido entre o ensino e a aprendizagem. Tradução de D . V. Moraes, Porto Alegre, RS: Artmed Editora.

Diogo, R. C.; Osório, A. S. & Silva, D. R. R. (2007). A Teoria Antropológica do Didático: Possibilidades de Contribuição ao Ensino de Física. In: *Anais do VI Encontro Nacional de Pesquisa em Educação em Ciências*, Florianópolis, SC.

Duval, R. (2016). Questões epistemológicas e cognitivas para pensar antes de começar uma aula de matemática. Tradução de M. T. Moretti. *Revista Eletrônica de Educação*

Matemática, 11(2), 2 40.

Gal, I. (2005). Towards 'probability literacy' for all citizens. In A. Graham. (Ed.), *Exploring probability in school*: challenges for teaching and learning (pp. 43-71). Kluwer Academic Publishers.

Glen, S. (2003). *Dependent Events and Independent Events*. From StatisticsHowTo.com: Elementary Statistics for the rest of us!.

Magalhães, M. N. & Lima, A. C. P. (2005). *Noções de probabilidade e estatística*. Editora da Universidade de São Paulo.

Oliveira Júnior, A. P.; Souza, N. G. S. & Datori Barbosa, N. (2019). A resolução de problemas no ensino de estatística no ensino fundamental: contribuições da Teoria Antropológica do Didático e a Equivalência de Estímulos. *Revista Eletrônica de Educação Matemática*, *14*, 1-18.

Oliveira Júnior, A. P. & Datori Barbosa, N. (2023). Tasks about impossible random events: a pedagogical game as a teaching tool for the early years of elementary school. *Acta Scientiae*, *25*(6), 183-208.

Oliveira Júnior, A. P. & Lozada, A. O. (2023). A pesquisa sobre a utilização e criação de livros paradidáticos para o ensino de probabilidade nos anos finais do Ensino Fundamental no Brasil. *Revista de Ensino de Ciências e Matemática*, 14(4), 1-25.

Smole, K. C. & Diniz, M. I. S. V. (2010). *Matemática*: ensino médio. Saraiva.

Tavares, M. (2014). *Estatística aplicada à Administração*. (3. ed., rev. ampl). Florianópolis: Departamento de Ciências da Administração da Universidade Federal de Santa Catarina.