



Hermeneutic-phenomenological study of the teaching practice of polyvalent teachers

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Abstract: Studies in Math Education have, in the last twenty years, presented substantial results regarding the teaching methodologies used in elementary education. Thus, considering the context of the training of pedagogue teachers who teach Mathematics, we seek with this paper to present a study carried out with a group of polyvalent



teachers about their methodological understandings concerning the teaching of fractions. For that, we used the hermeneutic-phenomenological methodology for data analysis, considering the alectic process as an instrument of extraction and interpretation. As results obtained in this study, we highlight the cultural aspects that stress the methodological choice by the teacher, here named cultural crystallization, as well as the gap between the mathematical concept of fraction and the methodological choice for its teaching. In addition to the results obtained through the hermeneutic-phenomenological analysis, the present paper brings possibilities and limitations regarding its use in research in Math Education.

Keywords: Fractions. Teacher Practice. Math Education. Hermeneutic. Phenomenology.

Estudio hermenéutico-fenomenológico de la práctica docente de los profesores pedagogos

Resumen: Los estudios en Educación Matemática han presentado, en los últimos veinte años, resultados sustanciales en cuanto a las metodologías de enseñanza utilizadas en la educación primaria. Así, considerando el contexto de la formación de profesores pedagogos que enseñan Matemáticas, buscamos con este trabajo presentar un estudio realizado con un grupo de profesores polivalentes sobre sus comprensiones metodológicas en relación con la enseñanza de las fracciones. Para ello, utilizamos la metodología hermenéutica-fenomenológica para el análisis de datos, considerando el proceso alético como instrumento de extracción e interpretación. Como resultados obtenidos en este estudio, destacamos los aspectos culturales que acentúan la elección metodológica por parte del docente, aquí denominada cristalización cultural, así como la brecha entre el concepto matemático de fracción y la elección metodológica para su enseñanza. Además de los resultados obtenidos a través del análisis hermenéutico-fenomenológico, el presente trabajo trae posibilidades y limitaciones en cuanto a su uso en la investigación en Educación Matemática.

Palabras clave: Fracciones. Práctica del Profesor. Educación Matemática. Hermenéutica. Fenomenología.

Estudo hermenêutico-fenomenológico da prática docente de professores polivalentes

Resumo: Os estudos em Educação Matemática vêm, nos últimos vinte anos, apresentando resultados substanciais acerca das metodologias de ensino utilizadas nos anos iniciais da educação básica. Assim, considerando o contexto da formação de professores pedagogos que



ensinam Matemática, buscamos com este trabalho apresentar um estudo realizado com um grupo de professores polivalentes acerca de suas compreensões metodológicas frente ao ensino de frações. Para tanto, utilizamo-nos da metodologia hermenêutico-fenomenológica para análise de dados, considerando o processo alético como instrumento de extração e interpretação. Como resultados obtidos neste estudo, destacam-se os aspectos culturais que tensionam a escolha metodológica pelo professor, aqui nomeados por cristalização cultural, bem como a lacuna entre o conceito matemático de fração e a escolha metodológica para seu ensino. Para além dos resultados obtidos por meio da análise hermenêutico-fenomenológica, o presente trabalho traz possibilidades e limitações quanto ao seu uso na pesquisa em Educação Matemática.

Palavras-chave: Frações. Prática Docente. Educação Matemática. Hermenêutica. Fenomenologia.

1 Introduction

Rethinking teaching practices in the second decade of the 21st century is characterized as a fundamental task for the development of pedagogical actions aimed at effective learning by students of the "new" contemporary world, that is, those who experienced the period of the Covid-19 pandemic.

In this sense, we could point out, at first, that if the present research was carried out in an atypical context, in which technology consisted of the only possible channel for the continuity of the educational process, such circumstance only evidenced, however, the establishment of reality already present in human experience, permeated by virtualization, by the insertion of digital technologies in everyday social life, since currently a large part of the world's population is thus connected, which means that all essential aspects of life are virtualized. Therefore, a reality was created that was no longer parallel, but rather concurrent with that understood as "physical", that is, a virtual reality.

Regardless of the educational level at which one operates, considering the way in which contemporary learning takes place is necessary since the "new students" have their own language and are continually in transformation, taking semiotics as a semiotic basis. signs produced by the real or digital world they experience.

Given this context, we pose the following questions: (i) regarding teaching methodologies, what would be the impacts suffered by polyvalent teachers in this scenario of such new characteristics, such as the advent of social digitality? When we assume the area of Mathematics and the promotion of its teaching in the early years of elementary school, how should those develop their practice? Is there any cultural device, from the point of view of the social operation, that influences the development of teaching work through the results that will come from internal and external evaluations of learning about Mathematics?

Thus, answering the aforementioned questions is characterized, a priori, as an invitation to understand how this new reality, as well as the results that will come from it, strain the teaching and learning of Mathematics in the primary education.

So, with this article, we seek to present a study about the methodological understanding of the teaching of fractions by pedagogical teachers and their relationship with culture, considering as a context a training course for pedagogical teachers working in the early years of an elementary school in public school systems.

For that, we used the hermeneutic-phenomenological analysis proposed by Alvesson and Sköldeberg (2009), through the so-called alectic process. To make the presentation of this



work clear, in the first part we bring the theoretical contributions that guide us about culture and teaching practice, as well as the contributions of the hermeneutic-phenomenological theory in research in Education. In a second moment, we discuss the method of data production and analysis, as well as the interpretations obtained through the alectic process.

2 The culture and teaching practice

The notion of pedagogical practice brought by the studies of Franco (2015) and Mometti (2021a) contributed to the understanding of teaching work as a set of actions intentionally thought out by the teacher, systematized by a methodological path and, above all, based on necessary elements in what refers to the [teacher's] knowledge for their development.

These elements are epistemological, praxiological, and ontological in nature. This means that for a teacher to exercise his function, he will have to have, simultaneously, the necessary contents to support his intentionality in the process, plans, and routes based on his previous experiences, and on what comprises his conscience of being social.

In this way, when we allude to the necessary knowledge for the teacher, moreover, as Tardif (2012) guides us, we can organize them into (i) practical, (ii) academic and training, and (iii) experiential experienced.

On the other hand, considering digital humanity and the social transformations resulting from it, we add mediated knowledge to the previously mentioned knowledge, as Mometti (2021a) brings us. With this new knowledge, necessary for the development of any type of teaching with the aid of technological and digital resources, we open space for new possibilities of study and research, since from the methodological point of view the teacher will not be able, without a shadow of a doubt, disregarding that their students belong to a completely digital society.

Nevertheless, through pedagogical practice, we can establish one of the observation points of the subject's formation process, regarding the formalization of socio-historically constructed knowledge by humanity.

This means that even though there are other ways of training — in the paideutic sense of the term — it is through the teacher's intentionality that the training path will have meaning and purpose in a specific social group. As an example, we can cite the historical development of teaching from individual tutoring, aimed at professional and completely technical teaching.

Thus, throughout history, Education — now understood as an institution — has come to act as an instrument for *enculturating* subjects and passing on what should or should not be practiced, always considering a specific purpose.

It should also be noted that with the above we mean, respectively, that there will be the *construction* and subsequent *crystallization* of the culture in which the learning subject is inserted, because through it the individual takes his beliefs, values, attitudes, and standards to another within their social group, especially when we consider language and coexistence.

In this way, we understand by cultural crystallization the process of reproduction and transformation, concomitantly, of the standards and values incorporated through social action, as highlighted by Sewell Jr. (2005). For example, a teacher using the argument "I teach this way, because I learned this way" shows, in his speech, the *cultural crystallization*.

This culture, understood from the perspective of Sewell Jr. (2005), is then internalized and, with the frequency of a given behavior, it crystallizes in what cognitive theory usually calls learning.



The teacher, when resorting to the knowledge cited by Tardif (2012), during his teaching, activates the contents available in the envelope of his culture, which recurrently occurs through the manifestation of what he brought from his experience and learned throughout his life.

Furthermore, his sociological awareness is manifested in his work method, which can be evidenced by the presence of the statements "I learned like this, I teach like this...", "When I was at school, I saw it like this, it was easier!", or still, "the student needs to respect the teacher, because in my time it was different".

3 The fractions teaching in primary education

According to Campos, Magina, and Nunes (2006), Friz, Sanhueza, Sánchez, Belmar, and Figueiroa (2008), Proença (2015), Ubah and Bansilal (2018), fractions are understood by polyvalent teachers in the early years with different meanings, a diversity that corroborates the idea, previously exposed, that the view brought from the teacher's previous experience can determine his understanding of a given mathematical concept. In this sense, the first important point to be highlighted is precisely the fact that such teachers have a polyvalence character in their teaching work, therefore not having in-depth training in this specific area.

Thus, according to Campos et al (2006), the understanding of the concept of fraction goes through five different meanings, which are: (i) fraction as a number, (ii) fraction as the relationship between part-whole, (iii) fraction as measure, (iv) fraction as quotient and (v) fraction as multiplicative operator. Each of these meanings, according to the authors, is worked in an integrated or isolated way, not following a specific order or pattern.

This tells us that the polysemy of the concept of fractions, for example, can be a first indicator of the need for teachers to know the mathematics they teach in more detail. Because, according to Ubah and Bansilal (2018), in the same class, the teacher can work, with the same method, the different meanings of fractions listed above. This, in any case, would lead to confusion of meanings, as well as possible difficulties generated in students regarding the construction of this knowledge.

Therefore, we are directed to the point of the debate about the need — or not! — for teachers' disciplinary specialization, especially when we consider the first years of elementary school, as indicated by Julio and Silva (2018).

In any case, the authors deal with the understanding that the absence of specific Mathematics disciplines in Pedagogy courses makes the process of incorporating mathematical concepts even more difficult, as well as the ways in which this discipline should be worked on. in the early years. In addition, as they also highlight in their work, those who opt for the Pedagogy course do so because they consider the large area of Exact Sciences something unattainable and, emotionally, impossible.

At this point, a question arises that the relevant literature would need to address: *if future pedagogues do not like mathematics, how can they teach it in primary education?* At first, such a question may seem simple to us, but it holds many points that we should investigate about the training of teachers who teach mathematics.

4 The hermeneutic-phenomenological theory and Math Education

Educational research in Math Education in Brazil especially focused on the interpretative construction of data extracted from video, audio, and interview recordings, questionnaire responses, and documented historical works, using the theoretical-



methodological contributions proposed by phenomenology with a strong hermeneutic inclination, since the last decades of the 20th century, as showed by studies by Bicudo (1999), Klüber and Burak (2008) and Kluth (2020).

In this way, combined with the effort to develop a set of methods that would help in studies carried out in so-called "naturalized" environments, such as the classroom and the school, for example, the so-called hermeneutic-phenomenological theory assumes as its initial premise the establishment of a course, on the part of the investigator, that aims at the reconstruction of the facts and/or phenomena studied using the elements that constitute it. This means that both the story and the context in which the researcher is involved contribute to a hermeneutic-phenomenological analysis.

About current research in Math Education, developed internationally from 2010 to 2020, according to the topic at hand, there are studies such as that by Barth-Cohen, Little and Abrahamson (2018), about the interpretation of recordings of in-service teacher classes.

According to the authors, "although in-service teacher education using video is gaining acceptance, little is known about what can be achieved with teacher trainees who are just beginning to understand student learning" (Barth- Cohen *et al.*, 2018, p. 3). What stands out, whether in the use of videos as a source of data or in the speeches enunciated in an interview, is precisely the *interpretation*.

Furthermore, in the citation, Barth-Cohen *et al.* (2018) show us the importance of the record, not only as a simple form of documentation, since through imagery and written records we are able to reconstruct the studied context and, above all, extract information that can contribute in a systematic and substantial way to the formative process.

Thus, by emphasizing that "little is known" with the records made, of teachers in their initial careers, Bath-Cohen *et al* (2018) make it clear that the interpretative methodologies still need to establish more objective assumptions for the analysis process.

Complementarily, there is the study by Edmonds-Wathen (2019) about *mathematical linguistics*, which deals with the construction of a theoretical-methodological framework that aims to interpret not only the language used to describe a certain mathematical discourse, but also the modus operandi that different languages employ when using mathematical terms in their speech.

In this sense both Edmonds-Wathen (2019) and Barth-Cohen *et al.* (2018) contribute to an expansion of what the so-called Critical Math Education has discussed in recent years in Brazil: bringing interpretations that place the socio-historical tradition of the subject together with unfounded methodological objectivity, above all, in solely ideological and subjective aspects of the subject. investigator. All the authors participate in a methodological movement resulting from the phenomenological school of the late 19th century, which emerged with the German mathematician Edmund Husserl. According to Josgrilberg (2015, p. 5), "Husserl discovered the possibility, through the idea of intentionality, of exploring the path of the meaning of things in their immediate manifestation in perception".

Thus, still according to the cited author, the understanding of intentionality meant for Husserl,

the overcoming of skepticism through the ability to leak all the ideological and cultural thicknesses that surround us and that give us objects already interpreted in systems such as language, worldviews, ideologies, habits, and all the cultural thicknesses that contaminate historically the original data. Intentionality describes an



aspect of the human spirit of going to the object, aligned with language, but not conditioned by it when it reaches the object in the original perception (Josgrilberg, 2015, p. 6).

Moreover, it was due to the notion of intentionality that the distinction between interpretation and *doxa*, opinion itself about a given phenomenon and/or object studied, takes a position in what becomes the phenomenological theory. In this sense, phenomenology is born with the concepts of *immanence*, *transcendence* and *epoché*, in the Husserlian perspective. So, still according to Josgrilberg (2015, p. 6):

For phenomenology, meaning is an original ontological relationship, and an empirical explanation is not possible for it or through deductive methods of linguistics, for example. Sense is the Gordian knot of phenomenology. Phenomenology tries to elucidate our relationship with the meaning of things by describing how we arrive at it, how we think about it, especially through language, and how we analyze its structure. It is clear that for phenomenology, meaning has a pre-linguistic dimension, although the meaning is given in the position of language.

In the perspective of Husserl (1986), immanence is a concept inherited from Cartesian philosophy and deals with an epistemology of the subject, as if he were already attributed a portion of truth and understanding about nature and its phenomena. For Husserl (1986) imminence is closely related to transcendence, the latter relating what Cavalieri (2013, p. 38) emphasizes by

the object of knowledge [which, our emphasis] is not itself present in the act of knowledge and, therefore, according to the essence of the intentional relation of consciousness, we are faced with the fact that the respective cogitatio [rationality, emphasis added] implies that consciousness is the consciousness of something that is not itself. In a totally different way, one can speak of transcendence when one posits being present in person and its negation, being thought without being present as such.

Nevertheless, within Husserl's phenomenology, there is talk of a transcendence of imminence, considering not only the origin as a scenic stage of action but also the understanding and interpretation of the analyzed knowledge.

Finally, by $epoché^{l}$, Husserlian thought gives us a nod to what Cartesian metaphysics had already promoted with the so-called "evil genius", that is, the suspension of judgments and considerations of value that we assume *a priori* without knowing the object in its essence. Thus, we can say that the epoché is "a provisional suspension of deforming mediations (even those necessary for empirical knowledge). The epoché is intended to stick to the phenomenon in an exercise to recover the original formative movement" (Josgrilberg, 2015, p. 8).

Furthermore, in the same way, that Barth-Cohen *et al.* (2018), the authors Formenti and Jorio (2018) discuss a methodological approach focused on phenomenological interpretation, which they call "multiple visions, multiple voices", underlining it as a *dialogic methodology*.

However, the use of the term dialogic, for the authors, does not occur in the historicaldialectical materialist sense, as established by the Marxist theory of the 19th century, but, above all, in the sense of establishing communication between all those involved in the same process/

¹ In the literature about reflective methodologies, we find the expression "alectic process" to designate the movement of suspension of interpretations before knowing all the points/elements of the process. Thus, through the alectic process, an analytical-phenomenological "instrument" is defined on the data considered.



phenomenon.

Considering the study carried out by Formenti and Jorio (2018) with higher education teachers, their source of information was characterized by a set of reports produced in a "reflective" way during a training workshop, which aimed to develop the "reflexivity" of the research subjects (Formenti & Jorio, 2018, p. 8).

It should be noted that the sources of information, for most studies, are constructed from the observed phenomenon. This means that when we consider the classroom, for example, as the locus of study, and everything that happens in it as a phenomenon of a social nature, the ways we must "collect" information are video and audio recordings, notes from researcher's observations, interviews with the participating subjects, as well as other forms of records made by all those involved.

But what do Formenti and Jorio (2018) understand by *reflexivity*? In a way, they understand the term as a movement to describe something that happened, an event in which the individual was active and participant, with additions of learning obtained in other experiences. Such a conception is consistent with the perspectives of reflective research in social sciences discussed by Alvesson and Sköldeberg (2009).

It is also noted that in the methodological proposal of Formenti and Jorio (2018), the reflective movement consists of a reconstruction, which, in a way, resorts to the interpretation of a portion of the reality already experienced, which is in line with the studies by Barth-Cohen *et al.* (2018), as well as Tobin and Ritchie (2012).

Tobin and Ritchie (2012) propose, for the development of research with data obtained through records and documents, a contribution called multi-method by the authors. According to this perspective, to interpret in depth, the social phenomenon studied, such as the pedagogical practice of a certain teacher or the students' speech during a mathematics class, one must mobilize not only a form of method but a combination of them.

Thus, concomitantly with obtaining the recordings of a class, we can obtain the audio recordings of the teacher and the records of his blackboard so that, in a second moment, already in the process of analyzing the *transformed data*², we can triangulate the object in question by the content, by the form, the aesthetics, the movements of the subjects involved, the intensity of speech, among other elements that contribute to the interpretive process.

Given the above, we can say that the perspectives deal together with theoreticalmethodological aspects defined by Alvesson and Sköldeberg (2009) as reflective analysis methodologies. Reflexive in the sense that it requires the subject-researcher to mobilize historical-cultural aspects in the process of interpretation and reconstruction of the studied phenomenon.

In this sense, in the light of the *objectivity-subjectivity* dyad, which from the Enlightenment period directed investigative studies towards the common goal of modeling the phenomenon based on the search for observed patterns, reflective methodologies consider the tradition of differentiating between objective/subjective³ methods through the insertion of the

 $^{^{2}}$ The expression "transformed data" refers to one of the fundamental stages of the multi-method theory, which deals with the recognition and subsequent transformation of information into analysis data. So, according to Tobin and Ritchie (2012) considering the studies developed with teachers in service, for example, we would have the following steps: (i) delimitation of the context for collecting information, (ii) identification of the information collected following a pre-analysis guided by the research question-problem, (iii) transformation of information into data for analysis and, finally, (iv) the analysis itself.

³ This differentiation alludes to the opposition of the Romantic movement to the Enlightenment perspective since for the latter the objectivity of the study of the phenomenon was achieved through logos (rationality), while for the former we had, in addition



self, that is, as the core of its epistemological basis, the *Heideggerian* existentialist discussion appears as a central piece, in the way we interpret the world and relate to it.

The hermeneutic theory understood as the search to implement a method for interpretation based on the "holistic understanding" of the phenomenon, starts to contemplate educational studies insofar as the subjectivity of the individuals involved in the study is considered, as well highlights the discussion by Josgrilberg (2015) about the hermeneutic-phenomenological in Education.

It should be noted that the use of the hermeneutic-phenomenological compound word is characterized, in a way, as a methodology for data interpretation. From a historical point of view, hermeneutics stems from phenomenology, an area of study initiated by Husserl, passing through Heidegger and Gadamer.

In addition, its foundation as a method arises in a controversy at the beginning of the 20th century, which preached that to interpret the social phenomenon it was necessary to use methods closer to the so-called hard sciences, which contained completely "objective" forms of study and interpretation, established at an epistemological level. Thus, the issue of subjectivism based on interpretation/understanding is at the center of the discussion about the analysis of social phenomena.

About this Alves, Rabelo, and Souza (2014, p.182) point out that only "the rebirth of Anglo-Saxon analytical philosophy and sociological movements based on pragmatism and phenomenology, replaced the idea of understanding as a central theme in the theory society", leaving in the background the question of the methodological framework to be followed.

Thus, according to Josgrilberg (2015, p.9):

The hermeneutic extension of phenomenology opens a door for dialogue with other instances that are concerned with the cultural elaboration of meaning. The approximation of the eidetic or essential sense with the sense sedimented in the everyday world, in culture, or reviewed in the sciences is necessary for the phenomenological description to be effective in the different forms of concrete life that we live. A phenomenology of education that only describes the ideational movement of the essence of education is halfway there. To serve education, it needs the hermeneutic expansion.

5 Methodological design: defining the reality to be interpreted

To contemplate the scope of this work, which was to carry out an investigative study about the methodological understanding of the teaching of fractions, by pedagogical teachers, and its relationship with culture, we selected as a cultural context a training course for pedagogical teachers who work in the early years. It should be noted that in this investigation we understand the cultural context as the historical-temporal *locus* where practices and actions take place, as well as reproductions and cultural transformations within a structure (Sewell Jr., 2005).

Thus, the context was characterized by a continuing education course carried out with a group of pedagogical teachers in the second semester of 2020. This course was specific to Math Education and was part of the research schedule of *Educação Matemática nos Anos Iniciais:*

to the *logos*, the consideration of the mythos (subjectivity). The logos-mythos dyad characterized the history of scientific thought from the 18th century onwards in the West.



aspectos metodológicos do ensinar⁴ developed at the University of São Paulo, São Paulo, Brazil.

This course was organized to be developed in three months, totaling a workload of 60 hours. In addition to the activities carried out at a distance, through the virtual learning environment (LMLS), the participating teachers had synchronous classes on the topics selected for the pedagogical work, which dealt with teaching methodologies of (i) fractions, (ii) multiplication, (iii) division, (iv) use of learning objects and (v) decimal numbering system.

Thus, our sources of information for the study in question were: (a) the recording of the synchronous class on the methodology of teaching fractions, (b) the field diary of the researcher-trainer, and (c) excerpts from the speeches given by the teacher's polyvalent course⁵ participants. All teachers participating in this study signed an assent term for the treatment of information following the ethical regulations in Brazil.

Within the specific goals of that research, its objectives were: (i) to promote the theoretical-practical study of methodologies for teaching mathematical concepts aimed at the initial years of fundamental education, (ii) to develop mathematical language as a guiding principle of teaching pedagogical practice in the primary education, (iii) understanding the notions of method and methodological choice by the participating teachers and (iv) producing support materials and pedagogical use for the participating teachers.

In the first edition of the course, developed in 2020, participated 158 pedagogues' teachers, working in municipal and state education networks in five Brazilian municipalities in the states of São Paulo and Espírito Santo, Brazil.

As already highlighted, this article presents, among the topics selected for the training course, the one that dealt with fractions and their teaching methodologies. In this way, and considering the proposal initially presented, we chose the hermeneutic-phenomenological methodology for the analysis of the work, which appropriates the theoretical contributions on reflexivity mentioned above (Alvesson & Sköldeberg, 2009; Alves *et al.*, 2014). In this sense, the data extracted from the collected information were used to compose the stages of the so-called alectic process, as highlighted by Alvesson and Sköldeberg (2009).

According to the same authors, to develop a reflective analysis methodology, we need to follow some steps, which are represented in Figure 1, through the *hermeneutic cycle*.

The hermeneutic cycle is so called because, once the steps are performed, it returns to the same element of analysis. This procedure is the same one used for carrying out *exegesis*, e.g. The advantage of using this methodology in educational studies refers to the possibility of integrating all sources of information collected in their own context, which can be found in the literature under the name of naturalized research, as mentioned by Josgrilberg (2015). Thus, according to the author, "phenomenology does not invent or construct the intended thing; it is the very thing that manifests itself. It is not a question of superimposing our mental or psychological structure on the thing that manifests itself. The intentional object cannot be

⁴ This research was carried out in partnership and concomitantly with the thesis project developed at Concordia University (Canada), and has the following objectives: (i) to investigate the main methodological forms that polyvalent teachers use to teach mathematics in the primary education, (ii) to study those considered cultural operators that influence the methodological choice in the early years, (iii) identify the most difficult mathematical concepts for the pedagogical work in the focus group and, finally, (iv) develop continuing education courses, as well as materials to support practice teacher with regard to the teaching of mathematics in the primary education.

⁵ It should be noted that both the recording of the synchronous class and the interviews with the teachers followed the regulations provided for by Brazilians resolutions No. 466 of December 12, 2012, and No. 510 of April 7, 2016, both directing research with human beings and their aspects ethics, as well as Law No. 13,709 of August 14, 2018, which deals with the protection and use of personal data.



confused with psychological, historical, scientific, or cultural conditions" (Josgrilberg, 2015, p. 7).



Figure 1: Schematic representation of the hermeneutic cycle

Source: Adapted by Alvesson e Sköldeberg (2009)

Added to this is the fact of enriching the interpretation through interaction with the studied object itself. For this reason, it is important to carry out, in the hermeneutic cycle, a part/whole movement, that is, for each of the analyzed parts of the process, one must return to the considered whole since the sociocultural characteristics greatly influence the interpretation to be developed.

Thus, following the steps highlighted in Figure 1, we have (1) *Prior knowledge of the object to be studied*. In our case, the teaching of fractions in the early years and their relationship with the methodological knowledge of the pedagogue teacher. (2) *Search for cultural standards and values experienced by the subjects who participate in the analytical process*, a step that is consistent with the sociocultural factor of the subjects participating in the research. (3) *Development of the interpretative process through the elements given by the information sources*, carried out through the researcher's writing about what he is visualizing and understanding of the parts to the whole). (4) *Approaching a first understanding of the case study*, e.g., the ways in which the course teachers understood fractions and their teaching methods. (5) *Modification of the lived and analyzed reality through interaction with the object studied*, a stage that took place, in our course, through the discussions promoted in a debate, leading to a moment of reflection and reanalysis of the process.

Furthermore, once these stages are completed, the whole process is restarted, until all possibilities of interpretation of the analyzed case are exhausted. As stated in the objective of this work, and it is worth mentioning, the chosen case for study was the search for a relationship between culture and methodological aspects of teaching fractions in the primary education.

6 Data and analysis

As explained in the previous section, the analysis and interpretation of data followed the assumptions of the hermeneutic-phenomenological methodology, discussed by Alvesson and Sköldeberg (2009). Thus, the first element analyzed concerns the biographical and professional



data collected from teachers still in the enrollment phase of the continuing education course.

Of the 158 teachers who initially enrolled, however, only 45 would follow the course for the initial years since most of the enrolled teachers worked in Early Childhood Education.

For this reason, we decided to divide the course into two modalities, one specific to the early years (the object of this study) and the other related to the teaching of mathematics in Early Childhood Education.

It was, therefore, 45, the number of polyvalent teachers integrating the modality focus of this work. Of this total, only 4 had completed their education in Pedagogy in person, following a regular course at an institution classified as a university according to the criteria of the Ministry of Education of Brazil in force in the year 2020⁶. The others already worked in the respective education networks, partners of the training course offered, and completed their training in Pedagogy at a distance, in a tutored way.

Another interesting fact that is worth discussing, about the participants, deals with the time off work as a polyvalent teacher in the early years, with more than half of the total already working in the profession for more than twenty years. This led us to realize, through the speeches of some of them during the synchronous moments of activity, that "college only served to give the diploma", and the practice and experience "already had".

This element evidence, at first, the presence of pedagogical security arising from the experienced practice. According to Sewell Jr. (2005), this interpretation is in line with the process of transformation and reproduction of culture within a structure, because as the action is incorporated by the individual, it becomes a pattern to be reproduced and, consequently, creates an idea of value, in the case mentioned, pedagogical security due to the time of professional activity.

Thus, in a second moment and following the hermeneutic cycle highlighted by Figure 1, the *anamnestic picture* previously presented gave us some initial inferences (I) for the beginning of the alectic process of analysis.

Such inferences followed the cultural contributions used as a basis for this work and were the following: (I1) The polyvalent teacher would work the fraction according to his school experience due to the value of pedagogical safety, (I2) The multipurpose teacher would not be too concerned with highlight/differentiate the meanings of fraction in your practice, (I3) The fraction would be understood, basically, in your practice, as the relationship of the whole with its parts.

Inferences 1 to 3 helped us in stages 2 and 3 of the alectic process, as highlighted in fig. 1. In this sense, a way to test such inferences — and their possibility of interpretative validation — would be by gathering other data obtained from other sources of information, such as the discussion recorded during the synchronous class and the collaborative forum on the theme of fractions.

The class analyzed for this study was organized didactically in the following topics: (i) understanding the concept of fractions, (ii) how a child learns fractions, and (iii) how we can teach fractions. The time spent for the whole class was two hours, with uneven distribution between the items mentioned. The information collected throughout the class helped us develop stages 3 and 4 of the alectic process.

During the presentation on understanding fractions (the first topic), participants in the

⁶ Criteria available at http://portal.mec.gov.br/sesu/arquivos/pdf/PadrDireito.pdf, viewed on March 10, 2022.



room were asked what a fraction was. The most recurrent answers were about the relationship between part and whole. As highlighted by Campos et al (2006) and discussed by Mometti (2021b), the understanding of fractions as part/whole is the most present mode when we talk about this content.

In a way, this is the simplest meaning, from a concrete point of view, to understand fractions, since we only consider parts that form an object, and how we would represent them when compared to this object. At this point, the presence of existentialist thought is notably perceived, emphasizing that "what I see and what I feel is what exists", therefore the fractions can be visualized through the gathering of parts into a complete whole.

Thus, resorting to the cultural contributions used as a reference for this study, we can note, in a complementary way, that learning fraction through the construction of this meaning is characterized as a point of crystallization, since that meaning passed through their school experience and was internalized as coherent and validated content.

In addition, this interpretation corroborates what was predicted by inferences I1 and I3, mentioned above, since one of the lines to respond to the questioning was the following:

The relationship part and whole. At least that's how I learned! — Teacher A (Ipsis litteris transcript of recorded speech, 2020).

Clearly, when we compare teacher [A]'s speech with the inferences and the cultural reference presented, we have no guarantee that all participants in that class thought the same way and, according to the hermeneutic-phenomenological perspective given by Alvesson and Sköldeberg (2009), generalizations are not the object of a reflective methodology.

However, analyzing the data registered in the collaborative forum on fractions, opened after this class in the virtual environmental of the referred course, we were able to verify that the crystallized culture, evidenced by the speech of professor [A], prevailed, and was central in the discussion about the ways to understand and teach fractions.

Thus, we could understand that the predominant meaning of fraction, for that specific group of teachers, is that of part/whole, assuming the understanding of fraction as a mathematical concept to be taught.

Therefore, I2 becomes valid, to complement this study, since not knowing the other meanings, as highlighted by Campos et al (2006), there would be no pedagogical concern to think of specific methods to teach such differences to children.

From this conclusion, we can perceive, in a third moment, that the methodological choice for a certain mathematical concept depends, almost unambiguously, on the degree of understanding of that concept. This means that, based on what was presented above, "I teach this way because this is how I came to know it and understand it".

In a moment after the speech, and dealing with the ways in which we represent fractions for students in teaching materials to support learning, a second teacher asked for the floor and commented the following:

But the drawing you used there in the question is not clear if it is the part painted blue or the part painted white. Hence, the child has no way of knowing unless you indicate this when talking about fractions. — Teacher B (Ipsis litteris transcription of the recorded speech, 2020)



As highlighted in the speech of teacher [B], the concern with the student's understanding of what will be taught is notorious, in addition to allowing us to develop an interpretation from the methodological point of view. Because, according to figure 2, which was presented during the class with the teachers, we could think that the part colored in blue would be the one in which the fraction will be represented. And it is precisely this "first" thinking that Husserl's epoché asks us to suspend.



Source: The Author

However, from the speech above we realize that there was a second interpretation of the teacher, mainly thinking about what her student would visualize. Thus, at that very moment, the following question arose: so, when we work with fractions with our students, do we use the idea of lack? Like the classic examples of pieces of pizza, cake, and chocolate?

After the launch of these questions, the debate was intensified among the professors participating in the course, which enabled us to develop the fifth stage foreseen by the alectic process.

Thus, the questions developed from that discussion promoted, in a way, a change in the pedagogical reality, and experienced, of those teachers, a fact evidenced by the following comment from a third teacher:

You know I never stopped to think about it. Generally, I teach with food, and painted squares at most. But, teaching as the remaining part, I had not even thought about it...— Teacher C (Ipsis litteris transcription of the recorded speech, 2020).

We can see, from the comment described above, that reflexivity was reached in one of the participants in the discussion, and this, according to the hermeneutic-phenomenological interpretation, promotes a change of reality, insofar as through reflection we are able to rethink our actions and, in this way, we change or interact sociocultural in a different way than would happen if we did not reflect.

In addition, to materialize that discussion and bring this reflexivity to more teachers, we requested as an activity the production of a lesson plan on fractions that did not contemplate the idea of absence. This proposal was launched to the participants as a task to be delivered in the virtual environment, and of the 45 participating teachers, 36 delivered the lesson plan.

Finally, the class ended with a debate about the fact that the idea of absence for the child is "easier" to be visualized, since, according to the speech of a fourth teacher:



A child always cries when he misses. That is why everything that is missing, and she perceives makes a difference. — Teacher D (Ipsis litteris transcription of the recorded speech, 2020).

This, again, leads us to the recurrence of the alectic process, validating the inferences initially proposed. However, it would be up to us to consider that when bringing the expression "easier" to the discussion, the teacher considers, in a way, the methodological aspect, since it is directly related to "how much a student can learn from that or that method".

Thus, the reflective process is evident in the teacher, especially in what we wanted to observe about the manifestation of questions associated with the method. Certainly, for this professor, the way of teaching a certain content reflects substantially on his own learning of the same, which gives us one more value, of the educational culture, crystallized, the *quality* associated with the method.

7 A hermeneutics of/for teacher training

As highlighted, with this work we seek to analyze, from the hermeneuticphenomenological perspective, a moment experienced as pedagogical teachers during a class on fraction teaching techniques, in a continuing teacher's training course.

At first, we highlight the carrying out of an anamnestic process with the participating teachers, considering the theoretical-methodological contributions provided by Alvesson and Sköldeberg (2009) about the hermeneutic cycle.

Thus, as presented throughout this work, the cultural crystallization revealed in the group of teachers studied revealed to us that the standards built around the teaching of fractions are configured by a teaching value totally directed towards a single practice, that is, to an understanding of fraction built from your experience.

In this sense, the highlighted speeches of the teachers, together with the assumptions given by the hermeneutic-phenomenological applied to the study, allowed us to perceive that the methodological notion of a given mathematical concept is closely related to conceptual understanding. This means, at first, that the analysis of the educational context allows us to reconstruct moments and thoughts on which our pedagogical practice is based.

Thus, verifying that the understanding of the participating teachers about the concept of the fraction was associated with the notion of part/whole, led us to the interpretation that the idea of method, for this group, as highlighted, depends on the crystallized culture resulting from professional experience about teaching fractions.

With this study, we were able to highlight some aspects concerning the pedagogical practice of polyvalent teachers regarding the teaching of fractions in the early years. In addition, we assume cultural contributions as an observation lens, highlighting culture as a set of beliefs, values, and internalized and crystallized standards, manifesting in the subject in the form of suggesting behavior, as Sewell Jr. (2005), that the structure itself — in the study in question, the classroom, and the pedagogical practice — influences the construction process of mathematical thinking.

Here, moreover, it is worth noting that the structural notion follows the theoretical guidelines of Giddens (2013), considering society not as a struggle between different classes, but rather as different constituent layers of a larger structure, which influence the action of individuals through the agency, just as the latter interferes in the structural disposition itself.

In a way, considering a classroom from a structural perspective does not lead us to close



our analysis view; on the contrary, it contributes significantly to the reflective methodology addressed by Alvesson and Sköldeberg (2009). In this sense, we can say, based on the aspects considered here, that reflexivity is a very important factor for the very structural survival of a classroom. This fact can be deepened through the considerations of Giddens (2013) about reflexivity and the *new modernity*.

However, how do such considerations contribute to the understanding of teaching practice in Mathematics Education? Faced with the data interpreted in the light of the hermeneutic cycle, through the alectic process, we noticed in the group of teachers studied that the load of the school experience lived greatly influences the way they understand fractions and teach them to children.

So, starting from this interpreted and understood result from a reflective-methodological point of view, we can think of continuing education courses, as well as disciplines for initial training, which consider more case studies and portions of "reality" for analysis by teachers, in order to in a way that leads them to develop their reflective thinking and perceive the cultural crystallizations present in their already incorporated mathematical conceptions.

On the other hand, we can additionally think of pedagogical and reading support materials that, when systematized, contribute to increasing the framework of possibilities for teachers, demystifying the discourse that "theory is one thing and practice is another". This is the central role of research, especially when of an educational nature: to show alternative paths and possibilities that contribute to teacher education, as is our interest.

Finally, we observed that the anamnestic chart carried out prior to the development of the class is in line with the results exposed by Gatti and Nunes (2009) about the training of pedagogues and the gaps in their initial training due to outdated curricula.

Furthermore, the existing gaps in the process of initial training of pedagogues, regarding the teaching of Mathematics, will only be resolved when carried out in a considerable curricular reform, a fact that can make use of the contributions of hermeneutic-phenomenological studies. Thus, our study opens paths for deepening and new possibilities for the training of teachers who teach Mathematics, whether initial or continuous.

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