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Concept Mapping to Measure Mathematical Experts' Number Sense

Mapeamento de Conceitos Para Medir o Sentido Numérico dos Especialistas em Matemática

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Abstract

The purpose of this study is to test whether concept mapping can be used for measuring the levels of number sense and use it to measure the mathematical experts' level of number sense. The sample included 39 undergraduate and post-graduate students of Departments of Mathematics in Greece. A paper and pencil test was administered to measure the level of number sense in different mathematical domains. Additionally, the participants were asked to create a concept map with $\frac{1}{2}$ as the central term. The results showed low levels of number sense with the majority of the participants responded in the number sense test by applying rules and algorithms rather than more holistic approaches that would indicate higher levels of number sense. Additionally, participants' performance in concept mapping was strongly related to their performance in the number sense test. Specifically, participants with low number sense scores tended to present poor concept maps.

Keywords: Mathematics education Number sense. Concept map. Fractions

Resumo

O objetivo deste estudo é testar se o mapeamento de conceitos pode ser usado para medir os níveis de sentido numérico e usá-lo para medir o nível do sentido numérico dos especialistas matemáticos. A amostra incluiu 39 estudantes de graduação e pós-graduação de Departamentos de Matemática na Grécia. Um lápis e papel de teste foram dados para testar o nível de sentido numérico em diferentes domínios da Matemática. Adicionalmente, foi pedido aos participantes para criarem um mapa de conceito c como termo central. Os resultados mostraram baixos níveis de sentido numérico e a maioria dos participantes respondeu ao teste do sentido numérico aplicando regras e algoritmos em vez de abordagens mais holísticas que indicariam altos níveis de sentido numérico. Além disso, o desempenho dos participantes no mapeamento de conceitos foi firmemente relacionado ao desempenho deles no teste de sentido numérico. Mais concretamente, os participantes com resultados de baixo nível de sentido tiveram a tendência de apresentar mapas de conceitos fracos.

Palavras-chave: Educação matemática. Sentido numérico. Mapa de conceitos. Frações.

1. Introduction

Mathematical literacy involves more than executing procedures. It comprises requisite mathematical skills that enable individuals to cope with the practical demands of everyday life (Cockcroft, 1986) and data needs of modern life (Steen, 2001). A mathematical literate person can estimate, interpret data, solve day to day problems, reason in numerical, graphical, or geometric situations, and communicate using mathematics (Ojose, 2011). Two aspects of meaningful mathematical experience are strongly emphasised in the literature: the development of an integrated knowledge base in mathematics and the communication of mathematical knowledge (National Council of Teachers of Mathematics [NCTM], 1989). Number sense is among basic mathematical skills that are essential for mathematical literacy (Steen, 2007).

Number sense is recognised as an important goal of mathematics instruction and has been established as one of the aspects to be covered in compulsory education, hence it appears in the curriculum of several countries where the mathematical activity is proposed as an activity that "makes sense" (Australian Education Council, 1990; NCTM, 2000). In the current study, the level of number sense developed by mathematical experts is tested using innovative tools that have been developed for the needs of the study. Additionally, the reliability of using concept mapping to measure number sense is tested.

2. Number Sense

Numbers most possibly cannot exist independently of quantities or symbols (Sophian, 2019). In that way, developing numerical sense as a process of studying relations between quantities and using symbols to represent these relations goes back to the birth of human race. Recent findings support the hypothesis that human brain intuit about numbers, or *numerosity* as Dehaene (1998) puts it. From very early on, before we even acquire language, develop number words and symbols, learn to count, and basically learn mathematics, we can distinguish less from more of something. In the years to come, developing the concept of number is an ongoing cognitive process which takes place in a richfull sociocultural environment. Students deal with numbers in a systematic way and develop number sense already from kindergarten.

Howden (1989) describes number sense as "*good intuition about numbers and their relationships*" (p.11). Number sense refers to a person's general understanding of numbers and operations and the ability to handle situations that include numbers. This ability is used to develop flexible and efficient strategies (including mental computation and estimation) in order to cope with numerical problems, reasonable judgment, and evaluation of the results, both in the mathematics classroom and in real life situations (McIntosh, Reys & Reys, 1992; Yang, 2008). Relatively close to the previous definition is the one provided by Sowder (1992) who described number sense as an organised conceptual network which allows relating numbers, operations, and their properties to solve numerical problems in a creative and flexible way.

The term "number sense" has also been used by cognitive scientists to describe the ability to represent non-symbolic quantities, which is present in animals and humans since very early stages of development (Feigenson, Dehaene & Spelke, 2004; Wilson & Dehaene, 2007). While mathematics education researchers approach number sense as a mathematical skill, cognitive scientists define it as an ability that appears in humans well before formal instruction. In this study, the mathematics education approach to number sense is adopted and used.

Based on previous literature, Reys et al. (1999) identified seven main components of number sense, which are also used in the current study: 1) Understanding the meaning of numbers 2) Recognizing the relative and the absolute size of the magnitude of numbers 3) Using benchmarks 4) Being able to compose and decompose numbers 5) Using several representations of numbers and operations 6) Understanding the relative effect of operations 7) Developing appropriate strategies and evaluating the reasonableness of an answer. Specifically, a main criterion for a number sense-based strategy is whether one or more number sense components appear in the solution process followed by the learner. Number sense is a way of thinking that often represents flexibility, inventiveness, efficiency, and reasonableness (Dunphy, 2007). Also, number sense is a holistic conception of quantities, numbers, operations, and the relationship between them, which should be efficiently and flexibly applied to daily life situations (Yang & Wu, 2010).

The importance of cultivating number sense in school mathematics has been highlighted by many reports (NCTM, 1989, 2000; Markovits & Sowder, 1994). According to McIntosh, Reys & Reys (1992) number sense "*reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics have certain regularity*" (p.3).

3. Testing Number Sense

Despite its importance in gaining deeper mathematical literacy, research on adults' level of number sense, which mainly focuses on elementary school teachers or pre-service teachers, has provided evidence for underdeveloped number sense (Yang, Reys & Reys, 2009; Tsao, 2004). For example, Alajmi & Reys (2007) designed an interview study to test teachers' appraisements of the reasonableness of answers provided by students and the ways they evaluated number sense, both considered as main characteristics of number sense. The results showed that the participants approached reasonableness by applying standard algorithms and finding the exact results, rather than counting on more holistic approaches that would indicate higher levels of number sense.

In the same line, research on elementary school pre-service and in-service teachers (Yang, Reys & Reys, 2009; Tsao 2004; Sengul, 2013) as well as on secondary school pre-service teachers (Almeida, Bruno & Perdomo-Diaz, 2016) indicated the extensive use of rule-based methods and algorithms as the main strategies applied, even though the participants were asked not to use written computation algorithms. Additionally, interestingly enough, the secondary school pre-service teachers counted more on algorithms and rules than the pre-service teachers of elementary school (Almeida, Bruno & Perdomo-Diaz, 2016). On the other hand, interviews on a small group of secondary teachers in Greece showed that the participants could use certain kinds of strategies in answering rational numbers estimation tasks which anticipated characteristics of number sense (Hadoglou, 2018).

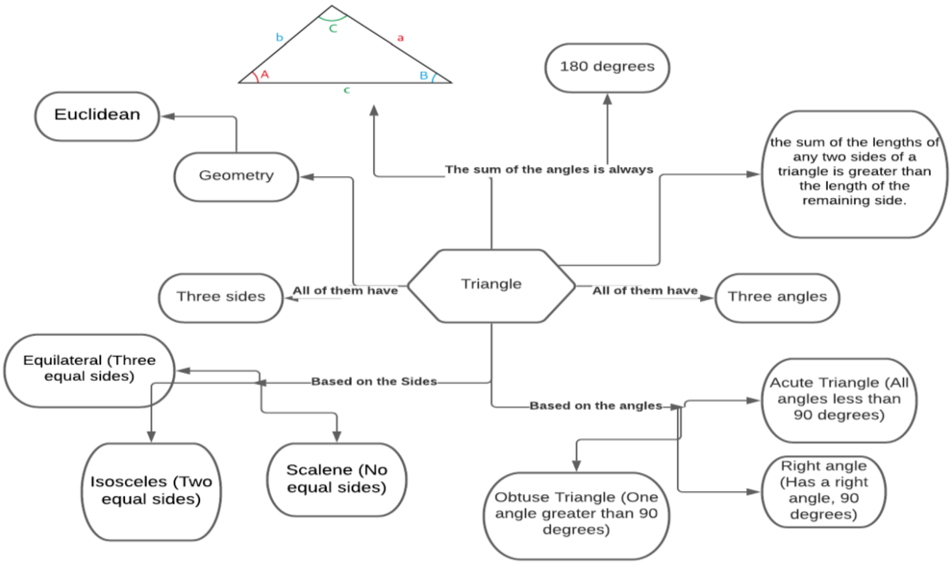
Taking under consideration the characteristics of number sense as presented above, it is quite a challenge to find a way to uncover participants' ability for deep conceptual thinking in mathematics such as for having deep number sense, using an instrument that does not imply testing with right and wrong answers, while being open-ended in order to allow for rich and detailed responses and being reliable and consistent at the same time. In previous studies, mathematical tasks have been administered to the participants that required the use of mental computations and estimation strategies in order to come to a correct response, in the form of open-ended questionnaires, that were usually administered with individual interviews (Alajmi & Reys, 2007; Almeida, Bruno & Perdomo-Diaz, 2016; Hadoglou, 2018; Şengül & Gülbağcı, 2012; Sengul, 2013). In the current study, the use of concept maps as a tool for measuring levels of number sense is also tested. We have reasons to believe that concept maps could be used as an alternative way to measure certain aspects of number sense.

4. Concept map and concept mapping

Concept maps were initially presented in 1972 by Joseph Novak at Cornell University as a way of "*determining how changes of conceptual understanding were occurring in students*" (Novak 1990, p. 937). Concept maps are a representation of meaning or ideational framework specific to a knowledge domain (Novak, 1990). Novak & Gowin (1984) believed that concept maps should consist of an expanding hierarchy of concepts, organised under more general, inclusive concepts. Specifically, Novak (1981, p.3) described concept mapping as *"a process that involves the identification of concepts in a body of study materials and the organization of those concepts into a hierarchical arrangement from the most general, most inclusive concept to the least general, most specific concept."* However, more recent approaches showed that not every concept map has to be hierarchically constructed. For example, researchers from the semantic network tradition tended towards spider-maps, which are maps with a general concept in the center and links coming out, much like the spokes of a wheel (Williams, 1998).

In a concept map, related concepts are represented as nodes and the specific relationship between them is indicated by linking words that are written along the line connecting the nodes (Bolte, 1999). These nodes are not limited to words, but symbols such as +, - or Σ may also be used (Novak, 1990). To put it simply, concept map is a diagram developed around a given concept. Every linked concept with the given one is placed in a node. The connection between the nodes is done with the lines called *linking lines*. Above every line, a word or a phrase explaining the way that the concepts are related to each other may exist. Figure 1 presents an example of a concept map of the term *Triangle,* constructed by the authors.

Figure 1. Example of concept map of "Triangle"



Novak and Canas (2006), describing the procedure for constructing a good concept map, argued that first it is important to define a domain of knowledge very familiar to the person constructing the map. The context can be well defined by a good central term that specifies the problem or the issue the concept map will help to resolve. The central term may lead to deeper and more essential thinking; a good central term may lead to a good concept map (Derbentseva, Safayeni & Canas, 2007).

5. Concept mapping as research tool

Concept maps are a very important and flexible tool to use both in learning and instruction-giving. The use of concept maps as an indication of connectedness of knowledge was based on Novak and Gowin's work (1984) in the field of science education. In the process of creating a concept map, students are engaged in a metacognitive activity that shapes and modifies their understanding of what they know when the map is constructed (McGowen & Davis, 2019; Hansson, 2005). Concept mapping, when used in conjunction with educational strategies, has led to superior learning achievements with students' thinking becoming more strategic and less associative (Novak, 1990; Ritchhart, Turner & Hadar, 2009). Moreover, concept maps have been suggested to be used as an evaluation tool for uncovering students' thinking about thinking and distinguishing deep from surface learning (Ritchhart, Turner & Hadar, 2009). Concept maps were "*developed to tap into a learner's cognitive structure and to externalise, for both the learner and the teacher to see, what the learner already knows*" (Novak & Gowin, 1984, p. 40). Consequently, concept mapping may act as a powerful methodology for uncovering students' conceptions in a way that is accessible both to teachers and students (Baroody, Baroody & Coslick, 1998).

Recent studies have successfully applied concept mapping for testing mathematics competencies in different mathematical domains and different age and expertise groups (Plotz, 2019; Conradty & Bogner, 2012; Ozdemir, 2005). For example, Hough et al., (2007) showed that by getting involved with concept mapping procedures, teachers reflected on their algebra knowledge and as a result they developed higher subject matter knowledge, as it appeared in the breadth, depth, and connectivity of their final maps. Park and Travers (1996) used concept maps as a comparative research tool, contrasting the maps of students with those of an “expert.” Also, concept maps were used as a research tool to investigate community college students’ mathematical knowledge within a narrow range of competence (Laturno, 1994) and as an instructional tool to document changes in the nature of middle school teachers’ thinking about their assessment practices (Wilcox & Lanier, 2000). In the same line, Wright (2008) asked pre-service teachers to construct concept maps for the term *fraction* to investigate their knowledge of fractions. The results showed that part-whole was the most frequent interpretation of fractions while none of the participants interpreted fractions as an operator (Wright, 2008).

The literature on concept mapping has thus provided evidence that its use as a research tool is valid and robust (Miller et al., 2009; Varghese, 2009; Williams, 1998; Plotz, 2019). Consequently, concept maps are suggested as a tool to measure knowledge of students across ages and levels of competencies (Mintzes, Wandersee & Novak 2001) with many researchers highlighting their efficiency on providing a visual representation of students’ knowledge (Conradty & Bogner, 2012). Concept maps have also been used as a means of documenting change in knowledge as a result of instruction (Hough et al., 2007; Williams 1998; Brakoniecki & Shah, 2017) and they are recommended as a means of assessing change. In this line, Williams (1998) explored the use of concept maps as a research tool in mathematics, particularly as the maps reflect conceptual understanding. His findings indicated a significant difference between students’ and experts’ concept maps and concluded that concept maps may provide important information about conceptual understanding.

In this study, concept maps were used as an assessment tool for number sense. To evaluate participants’ concept maps, they were compared with an *ideal* concept map as created by experts in mathematics, following the methodology suggested by Ruiz-Primo, Shavelson and Schultz (2001), Williams (1998) and Wright (2008).

6. The current study

In this study, the level of Number Sense in mathematical experts was tested using two measures: A Number Sense Questionnaire and a Concept Map Task. The first hypothesis is that, since the participants are considered to be experts in mathematics, they would present high level of number sense (Hypothesis 1) in both measures. It was predicted that the majority of participants would perform high in both tasks and the strategies they would develop for answering the Number Sense Questionnaire would be based on number sense components. It was also tested whether a Concept Map Task can be used for measuring the level of number sense. The participants were asked to design a concept map for the term $\frac{1}{2}$ following some basic instructions. The specific central term was chosen because it is a number with multiple interpretations and thus could allow for rich concept maps. It was expected that students’ responses in the two measures mentioned above would be related to each other, indicating that concept maps can be used to evaluate the level of number sense (Hypothesis 2).

7. Methodology

7.1 Sample

The sample included 39 undergraduate and post-graduate students from different Departments of Mathematics in Greece. Specifically, 15 of these participants were post-graduate students and the rest were undergraduate students at the beginning or the end of their studies. Independently of the different institutes that the participants attended, they all followed a very similar Mathematical Studies curriculum. Specifically, in all Greek Departments of Mathematics, the studies focus on five main mathematical sectors i.e., Algebra, Calculus, Geometry, Statistics and Computer Science. Mathematics Education is not a compulsory course; however, all the undergraduate participants had attended such a course. Considering the post-graduate participants, they all attended the same three-semester full time post-graduate program in mathematics education.

7.2 Materials

The aim of the study was not to test whether the students of Mathematical Departments were able to answer the given questions correctly or not, but to explore the way they approached the given solutions. Therefore, it was used certain tools that could reveal the strategies followed by the participants. The methodology used adopted characteristics from qualitative and quantitative approaches. Qualitative part was based on Thematic Analysis (Terry et al., 2017). Specifically, initially we familiarized with the data from the responses, then we created the categories in association with the adopted theoretical framework, and finally the responses were categorized and coded. Quantitative analysis of the categories was followed and differences between the categories were tested.

Two instruments were used to collect data for this study. The first was the Number Sense Questionnaire (NSQ). The NSQ was composed of 8 items (see Appendix) specifically designed to reveal the existence of one or more of the seven main components of number sense as presented in the introduction. Here, a brief description of the tasks is presented. Items 1 to 5 were developed by Yang, Reys & Reys (2009). In Item 1, participants were asked to compare decimal numbers with fractions without converting them from one form to the other, i.e., walking $\frac{7}{29}$km is longer distance than 0.4828km? In Item 2, they were asked to compare the fractions $\frac{30}{31}$ and $\frac{36}{37}$. In Item 3, they were asked to put the decimal point to the product 0.4975 x 9428.8 = 4690828. Item 4 was about comparing the ribbon we need to wrap two boxes, one in the shape of a cube and the other in the shape of a cylinder, with pictures of the two shapes provided. Item 5 was about proportions, and the participants were asked to compare two bottles of water with different volume and different price. In Item 6, which was designed by Alajmi & Reys (2007), the participants were asked to choose from a given list of alternates those numbers that make the inequality $3\frac{3}{8} \div \ldots > 4$ valid. Item 7 was about the density of the rational numbers and it was also used in former survey (Hadoglou, 2018) to examine number sense. The participants were asked to give any two numbers between $\frac{7}{8}$ and 1. In the last item, Item 8, which was used by Tsao (2004), the participants were asked to estimate if the product $\frac{21}{36} x \frac{7}{16}$ is bigger, smaller, or equal to $\frac{21}{64}$.

In Table 1, the relation between number sense characteristics and each item of the NSQ is presented in detail. For example, solving the Item 4 is considered to involve not only the use of several representations and operations, but also the development of appropriate strategies and to evaluate the reasonableness of an answer. A group of experts in mathematics education contributed on categorizing each item in each category of NS component.

The second instrument for data collection was the Concept Map Task (CMT). In this task, participants were asked to create a concept map in which the central term was the fraction $\frac{1}{2}$ with as many links as possible, with no restriction in the form of the provided links. This task was also used by Wright (2008), although with *fraction* as the central term.

Table 1. Relation between NS criteria and items

NS components	Tasks
Understanding the meaning of numbers	1, 6, 7, 8
Recognizing the relative and the absolute size of the magnitude of numbers	1, 2, 6, 7
Using benchmarks	1, 2, 3, 6, 8
Being able to compose and decompose numbers	2, 6, 8
Using several representations of numbers and operations	4
Understanding the relative effect of operations	3, 4, 5, 7, 8
Developing appropriate strategies and evaluating the reasonableness of an answer	4, 5, 6

7.3 Procedure

At the beginning, each participant was given the NSQ and was asked to complete it in the presence of the researcher. The following instructions were provided for the NSQ: “Answer the questions without making any written calculations and try to justify your answer with as many details as possible.” Enough time was provided for each item to be answered.

After the NSQ was completed by each participant, the CMT was administered to them, with only the central term provided in the middle of an empty sheet. None of the participants had ever created a concept map before, so they were all given specific instructions. Specifically, they were told that “concept maps are graphical tools for organizing and representing ideas. The concept map takes the form of a diagram that tries to define and explain the central term, which is in the center of the map. The concepts are usually enclosed in circles or boxes called nodes, they are connected to the central term and to each other with *linking lines*. The labels for the concepts may take the form of words, phrases, symbols, images, or any other representation that can relate in any way to the central term. Every *link* (arrows/lines) between the nodes may form a meaningful statement. To specify the relationship between two concepts/nodes, words may be used that can be placed above the linking line, which are called linking words or linking phrases. Another important characteristic of concept map is the cross-links. These are relationships or links between concepts in different segments or domains of the concept map. This means that the linking lines can start not only from the central term, but from any other node that appears on the map". An example of a concept map with "Triangle" as the central term (see Figure 1) was provided to the participants in order to help them understand what they are asked to do. There were no time restrictions for completing the CMT. All clarification questions were answered.

7.4 Results

Participants' responses in the NSQ were initially coded as Right or Wrong and the results are presented in Table 2. As it appears in Table 2, most of the participants responded correctly in the items of NSQ. Only in Task 6, in which they were asked to choose which of the given numbers would make the inequality $3\frac{3}{8} : ... > 4$ hold, less than half of the responses (35%) were correct.

Table 2. Frequencies and percentages of Right/Wrong answers in every item

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
Right	37 (95%)	32 (82%)	22 (56%)	29 (74%)	37 (95%)	14 (35%)	37 (95%)	28 (71%)
Wrong	2 (5%)	7 (18%)	17 (44%)	10 (26%)	2 (5%)	25 (65%)	2 (5%)	11 (29%)

The low performance in Item 6 can be due to the complexity of the item. Specifically, several number sense criteria were involved in this item (see also Table 1), such as understanding the meaning of numbers, using benchmarks, understanding the relative effect of the operations, developing appropriate strategies, and evaluating the reasonableness of an answer. The participants had to choose which numbers could make the inequality $3\frac{3}{8} \div ... > 4$ hold. The most common mistake was to select the numbers: $\frac{3}{5}$, 0.9 and 0.05 as correct answers. Interestingly enough, those responses show that participants appeared to have overcome the *division makes smaller* misconception, by applying the rule that dividing with a number smaller than 1 results in a number bigger than the dividend. However, from the given numbers, only $\frac{3}{5}$ and 0.05, (not 0.9) may result in a number bigger than 4. Thus, probably the participants failed this item because they overgeneralised the rule that the division with a number smaller than 1 results in a number bigger than the dividend, and they applied it to all given numbers that were smaller than 1.

In the following analysis, we focused on the strategies used by the participants. Both correct and incorrect responses were coded based on the kind of the solution strategy applied. Specifically, the answers that showed one or more of the number sense criteria (as they appear in Table 1) were categorised as "Number Sense" (NS). The responses that combined the use of a number sense criteria together with algorithms or memorised rules were categorised as "Partial Number Sense" (PNS). The answers in which algorithms and memorised rules were applied were categorised as "Rule Based" (RB) responses. The answers in which there was insufficient or no justification at all, even if the answer was correct were categorised as "Incorrect Justification."

For example, those responses that compared the fractions $\frac{3}{8}$ and $\frac{7}{12}$ by converting them to decimal numbers were categorised as RB. On the contrary, those that used the number $\frac{1}{2}$ as benchmark were categorised as NS. Answers like $\frac{3}{8} = 0.375 < 0.5$ and $\frac{7}{12} > \frac{1}{2} = 0.5$ so $\frac{3}{8} < \frac{7}{12}$ were characterised as Partial NS. In this case, there is a combination of using $\frac{1}{2}$ as a benchmark and converting a fraction into a decimal number. Two experts on mathematics scored the responses using the above criteria and agreement between the two scorers was 97%.

The results of this coding for each task of the NSQ are presented in Table 3. The highest number sense performance appeared in Item 7, which was about the density of rational numbers. Specifically, the participants were asked to find two numbers between $\frac{7}{8}$ and 1. The most common strategy was to convert the given number to equivalent fractions like $\frac{28}{32}$ and $\frac{32}{32}$ and report the numbers $\frac{29}{32}, \frac{30}{32}, \frac{31}{32}$.

As appears in Table 3, the lowest number sense performance appeared in Item 8, where the participants had to decide if the product $\frac{21}{36} x \frac{7}{16}$ is bigger, smaller, or equal to $\frac{21}{64}$. In this item, only 6 out of the 39 given answers were categorised as NS. In four of these cases the fractions $\frac{2}{3}$ and $\frac{1}{2}$ were used as benchmarks, *i.e.*,$\frac{21}{36} < \frac{2}{3}$ and $\frac{7}{16} < \frac{1}{2}$ so $\frac{21}{36} x \frac{7}{16} < \frac{2}{3} x \frac{1}{3} = \frac{1}{3} \approx \frac{21}{64}$. In another response that was also categorised as NS, the operand numbers were converted to equivalent fractions, and with the use of benchmark, the equivalent number was compared with the given, i.e.,$\frac{21}{36} x \frac{7}{16} = \frac{3x7}{3x12} x \frac{7}{16} = \frac{7}{12} x \frac{7}{16} = \frac{49}{192} < \frac{1}{3} \approx \frac{21}{64}$. In Table 3, the number of each category's answers per item is presented.

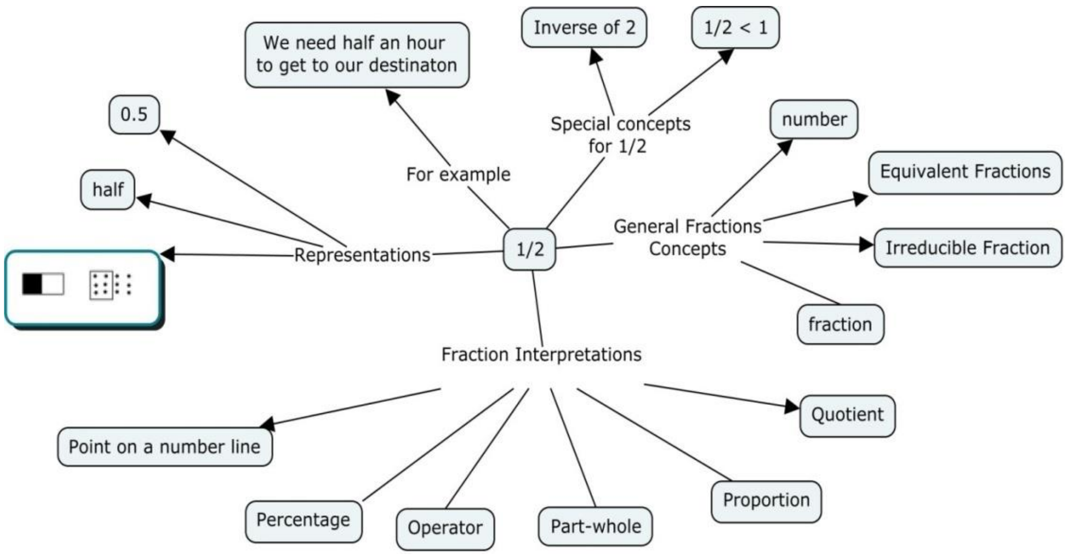
Table 3. Number of answers in every category per item

	NS	PNS	RB	Incorrect Justification
Item 1	24	2	8	5
Item 2	12	1	19	7
Item 3	18	0	14	7
Item 4	8	1	24	6
Item 5	17	0	20	2
Item 6	12	5	11	11
Item 7	28	3	4	4
Item 8	6	7	16	10
Total	125	19	116	52

7.5 Analysis of the Responses in the Concept Map Task

In order to analyze the responses in the CMT, the maps presented by the participants were contrasted with a prototypical map, created by the first author, with the help of other experts on mathematics (Figure 2). This *master map* consisted of sixteen nodes related to $\frac{1}{2}$, that could be placed in five main categories based on the characteristics of these nodes and their relation to the central term. Three nodes were categorised as "Representations", namely a Verbal (e.g., one over two), a Schematic (e.g., a picture of half a pizza or a relevant picture) or a Decimal representation (e.g., 0.5). The nodes that signified $\frac{1}{2}$ as a Proportion (e.g., 1 glass of water for 2 teaspoons of coffee), as a Quotient (e.g., 1:2), as Part-whole (e.g., one out of two or relevant picture), as Operator or as Percentage (e.g., 50%) or as Point on a number line were named as "Fraction Interpretations." Two categories of nodes were named as "Special Concepts for $\frac{1}{2}$," responses that presented any Inverse of 2 (e.g., $\frac{1}{2}$x 2 = 1 or any verbal reference to inverse) or responses including "is Smaller than 1" (e.g., $\frac{1}{2} < 1$). Four nodes were categorised as "General Fractions Concepts," namely a Fraction (e.g., verbal reference), a Number (e.g., verbal reference), an Equivalent Fraction (e.g.,$\frac{1}{2} = \frac{2}{4} = \frac{a}{2a}$) or an Irreducible Fraction (e.g., verbal reference). Finally, a node in the master's map contained an "Example with $\frac{1}{2}$."

Figure 2. Master's concept map



The participant would get one point for coding the participants' concept maps for each of those nodes that appeared in a map; repeating the same nodes in different formats would not add more points to the score. In Table 4, the frequencies of each of the possible nodes that appeared in participants' responses to the CMT are presented. As it appears in Table 4, the "Fraction" is the one with the highest frequency, whereas, interestingly, the "Operator" and "Point on a number line" was not mentioned by anybody.

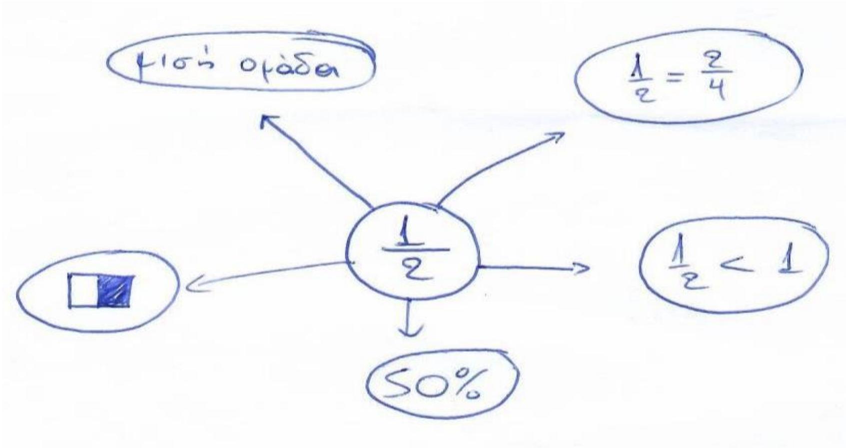
Table 4. Nodes frequency on concept maps

	Nodes	N=39
Representation	Verbal representation	28 (71.7%)
	Decimal representation	26 (66.6%)
	Schematic representation	15 (38.4%)
Fraction Interpretation	Quotient	15 (38.4%)
	Percentage	12 (30.7%)
	Part-whole	9 (23%)
	Proportion	4 (10.2%)
	Point on a number line	0 (0%)
	Operator	0 (0%)
General Fraction Concepts	Fraction	29 (74.3%)
	Equivalent fractions	20 (51.2%)
	Number	18 (46.1%)
	Irreducible fraction	1 (2.5%)
Special concept for ½	Smaller than 1	9 (23%)
	Inverse of 2	5 (12.8%)
Example	Example with $\frac{1}{2}$	18 (46.1%)
Total		209

None of the participants scored as high as 16, which is the maximum score based on the nodes that appear on the master map. The highest score achieved was 11, and the average score was 5.3 (*SD*=2.41), which is less than half the maximum score and thus, it could be characterised as moderate or even low, taking under consideration that the participants were experts in mathematics. An example of a concept map with a medium score is presented in Figure 3.

To test the second hypothesis considering the relation between the responses in the two measures that were used, performance in two tasks were compared to each other. The participants' responses to NSQ that were categorised as NS were coded as 2, PNS answers were coded as 1 and absence of response, false or intelligible responses were coded as 0. Using this coding, a performance to the NSQ was calculated for each participant. In the same line, performance in the Concept Map Task was measured by the numbers of nodes presented by each participant as demonstrated above. Pearson correlation test between these two variables indicated that, in line with our hypothesis, participants' performance in the CMT was strongly correlated with their performance in the NSQ $r(37)=.93$, $p<.01$.

Figure 3. An example of a concept map with medium score (5)
[translation from Greek: half unit]



7.6 Participants' Profiles

In order to test the way participants' responses in the Concept Map Task were related to the performance in NSQ in more detail, participants were classified in two categories based on a median split on NSQ scores: 22 participants were classified as Low NS (i.e., with NSQ score lower or equal to 6 points: *M*=4.6, *SD*=1.49, *min/max*=2/6) and 17 participants were classified as High NS (i.e., with NSQ score higher or equal to 7 points: *M*=9.9, *SD*=1.67, *min/max*=7/13). Most of the participants were categorised as Low NS. Table 5 presents the scores achieved in CMT and the number of participants who achieved this score per NS group. As it appears in Table 5, Low NS participants tended to attend low scores in the concept mapping task, and High NS participants tended to attend higher scores in CMT.

Table 5. Number of participants per NS group and the achieved scores in CMT

CM Score	1	2	3	4	5	6	7	8	9	10	11	Total
Low NS	2	1	6	5	2	1	4	1	0	0	0	22
High NS	0	0	0	1	6	2	2	1	3	1	1	17

7.8 Concept maps analysis per NS group

In the following analysis (see Table 6) how the different performing groups (i.e., High NS and Low NS group) performed in the concept map task was tested, by specifically testing the numbers of nodes in each special category of the fraction concept map (i.e., Representations, Fraction Interpretation, Special Concepts for $\frac{1}{2}$, General Fractions Concepts, Example with $\frac{1}{2}$). As appears in Table 6, with the only exception of the "Example with $\frac{1}{2}$", the participants with High NS presented higher percentages in each of the CM categories.

Table 6. Concept map Categories per NS group frequency

	High NS (N=17)	Low NS (N=22)
Representations (Number of representations: 3)	34, 66.6% (51, 100%)	35, 53.4% (66, 100%)
Fraction Interpretations (Number of interpretations: 6)	26, 25.4% (102, 100%)	14, 10.6% (132, 100%)
General Fractions Concepts (Number of concepts: 4)	36, 52.9% (68, 100%)	31, 35.2% (88, 100%)
Special Concepts for $\frac{1}{2}$ (Number of concepts: 2)	10, 29.4% (34, 100%)	5, 11.3% (44, 100%)
Example with $\frac{1}{2}$ (Number of examples: 1)	10, 58.8% (17, 100%)	8, 36.3% (22, 100%)
Total (ν=16)	116, 42.6% (272, 100%)	93, 24.6% (352, 100%)

Note. The numbers listed in parentheses, under the group name, are the maximum numbers of nodes that could appear in each category. For example, if all 17 participants with High NS performance presented all possible Representations, then there would be 17x3 = 51 possible nodes like that.

In what follows, possible associations are tested between the two different NS performance groups and each of the five categories of concept map's nodes they presented. All the tests presented are referred in Table 6. There is no statistically significant association between the performance group and whether they presented the fraction "Representations" in their map or not, $\chi^2(1, N_1=117) = 2.21$, $p>.05$. There was a significant association between the groups of NS performance and whether or not they provided "Fraction Interpretations" in their concept map, showing that High NS performing participants tended to provide "Fraction Interpretations" in their map, while Low NS performing participants did not, $\chi^2(1, N_2 = 234) = 8.99$, $p<.01$.

Additionally, there appeared to be a significant association between the performance group and whether they provided "General Fractions Concepts" in their maps or not, showing that High NS participants tended to include "General Fractions Concept" in their concept maps while the Low NS participants did not, $\chi^2(1, N_3=156)=4.91$, $p<.05$. Also, there was a significant association between the performance group and whether they provided "Special Concepts of $\frac{1}{2}$" in their maps or not, showing that High NS participants tended to include "Special concepts of $\frac{1}{2}$" in their concept maps while Low NS participants did not, $\chi^2(1, N_4=78) = 4.02$, $p<.05$. It appears that the High NS performance group presented higher frequency of "Example with $\frac{1}{2}$" nodes compared with those provided by the Low NS performers, but this difference was not significant, $\chi^2(1, N_5=39) = 1.94$, $p>.05$.

To sum up, there was a statistically significant difference between NS group and concept map categories concerning three out of the five categories, i.e., Fraction Interpretations, General Fractions Concepts, and Special Concepts of $\frac{1}{2}$. In those categories, the High NS participants tended to present richer concept maps than the Low NS participants. However, there was not a statistically significant difference between NS group and the other two categories of nodes in the concept maps, i.e., Representations and Example with $\frac{1}{2}$.

8. Discussion

This study tested mathematical experts' level of number sense, which refers to a person's general understanding of numbers and operations and the ability to deal with situations that entail number reasoning, which appear in the mathematical classroom or in real life situations (McIntosh, Reys & Reys, 1992; Howden, 1989; Sowder, 1992). A paper and pencil test, designed to test the existence of one or more of the seven main characteristics of number sense as defined in the literature (Reys et al, 1999), was administered to a sample of participants from Departments of Mathematics, either post-graduates or undergraduates. Additionally, the participants were asked to create a concept map with the fraction $\frac{1}{2}$ as the central term. This way, it could be tested whether concept maps could be productively used as a tool to measure number sense level or not. In the process of creating a concept map, students are engaged in a metacognitive activity that allows uncovering the participants' deep conceptual thinking (McGowen & Davis, 2019; Hansson, 2005; Conradty & Bogner, 2012). This is the main reason why it was expected that concept mapping the term $\frac{1}{2}$ would be correlated with number sense.

The results of the study showed low level of number sense, considering the level of expertise of the participants who are students in university departments of mathematics. The majority of them gave correct responses in the test that included different mathematical problems, showing their high level of expertise. However, their responses were based on applying rules and algorithms, instead of following the instruction of the researchers to avoid written calculations and algorithms. This finding is aligned with previous findings showing low level of number sense in adults, elementary and secondary pre-service teachers (Yang, Reys & Reys, 2009; Tsao, 2004; Almeida, Bruno & Perdomo-Diaz, 2016). Its appearance in the mathematical expert group of participants in the current study indicates the overinvestment of formal mathematics education in algorithms and rules rather than in the deep meaning of the concepts and their properties.

Interestingly enough, the only exception in which incorrect answers outperformed the correct ones was the task in which participants had to choose a number from a set of given alternatives that would make the inequality $3\frac{3}{8} \div \ldots > 4$ hold. The most common mistake was to choose all the given numbers that were smaller than one, including 0.9 and $\frac{3}{5}$, which results in an outcome smaller than 4. These results show that mathematical experts managed to disengage from the misconception that division always makes smaller, which affects students in a wide age spectrum (Christou, 2015), by using the rule that multiplication or division with a number smaller than 1 results in a number smaller or bigger than the other operand number respectively. This is in line with previous studies with mathematical experts (Obersteiner et al., 2015). However, the mathematical experts of our sample appeared to have overgeneralised this rule, ending up to different kinds of mistakes.

The results from the analysis of the concept maps created by the participants were quite the same. The majority of them responded extremely poorly in the specific task and presented concept maps of the term $\frac{1}{2}$ with less than 6 nodes, while the expert concept map included 16 nodes. Additionally, the average score was equal to just one third of the maximum possible. Different representations most appeared in the maps. Interestingly, none of the participants linked the specific fraction with "Operator" and "Point on a number line", showing that, as previous research has shown, understanding fractions as an operator is difficult even for experts in the field (Behr et al., 1997). Though multiplication and division are parts of the function of fractions as operators (Lamon, 2005), the researchers believe that the participants were referring to algorithmic operations with fractions rather than using a fractional quantity to perform an operation on another quantity. Our results are identical to those of Wright (2008), in which participants were asked to construct a concept map using as a central term the concept "Fraction", since none of the participants presented the interpretation "Operator". Moreover, "Point on a number line" was referred once in Wright (2008), while it was not referred by anyone in this study. Regarding number sense groups, participants who were categorised as High NS tended to present more Fraction Interpretations, General Fractions Concepts, and Special Concepts of $\frac{1}{2}$, while there was no statistically significant difference in the other two categories of nodes, i.e., Representations and Examples with $\frac{1}{2}$. In each of the concept map categories, Low NS participants tended to present fewer nodes in their concept maps than the High NS participants.

These results show that mathematics education, especially in its higher levels, overestimates the application of algorithms and rules, without giving the necessary attention to cultivating deep conceptual thinking about the mathematical concepts at hand. The participants of the study, who are considered mathematical experts and may become future mathematical teachers, appeared to have a very limited number sense and a poor repertoire of strategies to use in everyday mathematical problems which could be solved without written algorithms.

Additionally, these results present empirical support for the validity of concept maps as a tool for number sense measurement. The strong correlation between performance in the number sense questionnaire and performance in the concept mapping has provided additional empirical evidence that concept mapping may be used as a useful tool to measure number sense.

The main limitation of the study is the small sample of participants, which cannot allow safe generalization. Future studies using also individual interviews could provide additional information about both number sense and how students reason when getting involved in concept mapping tasks. Also, it would be preferred if the participants had prior experience in concept mapping and knew beforehand what they were going to be asked to do.

Despite the specific limitations, the results may support the claim that concept mapping may be used to measure number sense. As the literature has also shown, concept mapping may not only be used as an evaluation tool, but also as a teaching material (Novak & Gowin, 1984; Novak, 1990). The results of this study may offer the ground for suggesting using concept mapping as a teaching material to cultivate number sense. That is because, as also appeared in the current study, concept maps may bring to light the plurality of the concepts and representations that may be linked with specific concepts at hand, and thus, may reveal the depth of conceptual knowledge around a mathematical notion. However, more empirical research needs to take place, testing the characteristics of the learning environments that need to be built for the concept mapping to be assimilated in instruction. Also, teachers should be convinced about their usefulness and get proper training for applying them in their mathematical classroom.

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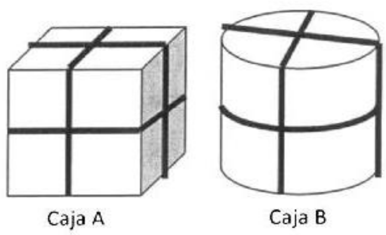
10. Appendix

Task 1 Place the numbers in order from the largest to the smallest $0.4828 , \frac{13}{38} , \frac{8}{15} , \frac{17}{16} , 0.966 , \frac{7}{29} ,$	**Task 5** A bottle of water of 600ml costs 18 cents and a bottle of water of 1500ml costs 35 cents. Which one is more profitable?
Task 2 Comparison of fractions $\frac{30}{31} , \frac{36}{37}$	**Task 6** Choose the numbers that make the inequation $3\frac{3}{8} : - > 4$ valid. $3\frac{3}{4} , 1.54 , 1\frac{1}{5} , \frac{3}{5} , 0.9 , 0.05 , 2.5$
Task 3 Find the place of the decimal point 0.4975 x 9428.8 = 4690828 a) 46.90828 b) 469.082 8 c) 4690.828 d) 46908.28 e) I can't choose without doing the exact calculation	**Task 7** Find three numbers bigger than $\frac{7}{8}$ and smaller than 1.
Task 4 We want to wrap two boxes with tape. Case A is a box in shape of cube with 10cm size. The height and diameter of case B is also 10cm. Which box need more tape? 	**Task 8** Without calculating the exact answer, choose the best estimation for: $\frac{21}{36} x \frac{7}{16}$ a) Bigger than $\frac{21}{64}$ b) Smaller than $\frac{21}{64}$ c) Equal to $\frac{21}{64}$ d) Impossible to choose without the exact calculations