USING TOUCHSCREEN DEVICES TO IMPROVE PLANE TRANSFORMATION IN HIGH SCHOOL CLASSROOM

USANDO DISPOSITIVOS TOUCHSCREEN PARA PROMOVER O APRENDIZADO DE TRANSFORMAÇÕES NO PLANO NA SALA DE AULA COM ALUNOS DO ENSINO MÉDIO

Alexandre Assis
profalexandreassis@hotmail.com
Federal Rural University of Rio de Janeiro, Brazil

Marcelo Bairral
mbairral@ufrrj.br
Federal Rural University of Rio de Janeiro, Brazil

ABSTRACT

Finding ways to improve geometry learning with dynamic geometric environment with touchscreen (DGEwT) is a new challenge in mathematics education. In this paper we illustrate some strategies concerning plane transformation used by two students to solve tasks on GeoGebra App and on Geometric Constructer (GC). Data came from videotapes of students working on the software, written answers for each task, the use of one shift in which he or she could write down and describe the function of each device icon, and from recorder SCR PRO. Two analytical protocols are presented. The proposed task and the possibilities of interaction provided by each device were fruitful to make emerge concepts related with plane transformation and to help students to solve the task by making spontaneous composition between them in a non-linear reasoning. Symmetry based on mirroring ideas took place in students' strategy as a scaffolding concept during their interaction.

Keywords: mobile device; learning; isometry; GeoGebra App; Geometric Constructer.

RESUMO

Encontrar maneiras para melhorar a aprendizagem de geometria em ambiente de geometria dinâmica com touchscreen (AGDcT) é um novo desafio no ensino de matemática. Neste artigo, ilustramos algumas estratégias relativas à transformação no plano usadas por dois alunos para resolver tarefas no GeoGebra App e no Geometric Constructer (GC). Os dados foram obtidos a partir de gravações em vídeos de estudantes trabalhando no software, respostas escritas para cada tarefa, o uso de um recurso no qual ele poderia escrever e reescrever a função de cada ícone de dispositivo, e do gravador de tela SCR PRO. São apresentadas análises de dois registros. A tarefa proposta e as possibilidades de interação proporcionadas por cada dispositivo foram frutíferas para fazer surgirem conceitos relacionados com a transformação no plano e para auxiliar aos alunos a resolver a tarefa, realizando composição de maneira espontânea em um raciocínio não-linear. A simetria baseada em ideias de espelhamento ocorreu na estratégia dos alunos como um conceito de andaimes durante sua interação.

Palavras-chave: Dispositivo móvel; Aprendizagem; Isometria; GeoGebra App; Geometric Constructer.

1. Introduction

As we have had a first major shift (cognitive and epistemological) and improved teaching by passing from paper and pencil environments to dynamic geometry environment (DGE) with drag and drop activities (e.g. Cabri Géomètre, Sketchpad, etc.), now we have a further shift and improvement with the transition to multi-touch environments (e.g. Geometric Constructor, SketchPad Explorer, Sketchometry etc.) and to the variety of simultaneous fingers' actions they allow. The evolution of digital technology makes available different practices in the classroom, specifically related to the way users can interact with the screen: from the drag and drop actions with the mouse to the tap, drag, and flick with one or more fingers on the screen of multi-touch devices and from the one-to-one interactions of the former to the multiple simultaneous interactions that the latter makes possible. These different technological features allow designing different tasks, which can change the cognitive processes of users and deeply modify their mathematical inquiries.

The way we deal and interact with touchscreen devices is providing new insights and challenges in mathematics learning and instruction (Arzarello et al., 2014). For instance, rotating and other kinds of gyrating movements on screen often take place, due the freedom of handling a touchscreen device. In this paper we discuss results from a research project[1] that investigates aspects of geometric learning during the process of solving tasks dealing with dynamic geometric environment with touchscreen (DGEwT). Particularly, we illustrate some strategies used by Brazilian High School students applying plane transformation concept to solve tasks on GeoGebra with touch and on Geometric Constructer (GC) software. The article summarizes geometric strategies created by students and provides reflection on how task designing can improve teaching practice with DGEwT.

2. Touchscreen as a new semiotic resource in mathematical thinking

Mobile touchscreen devices are extensions to our bodies in their sensorial, cognitive, emotional and social dimensions. In this expanding process our brain also will adjust to what is offered (Damásio, 2010) as the touches on screen are bringing new settings to the brain. Such extensions and adjustment bear implications to the way we can learn mathematics using DGEwT. We assume that touchscreen manipulation on a mobile device is not cognitively the same as mouse clicks, those we often do in dynamic geometry environment (Arzarello et al., 2014), for instance, due to the simultaneity of motion in different elements (points, sides, angles, areas etc.) from one picture (Bairral et al., 2015).

Mobile touchscreen devices provide more freedom in manipulation, that particular way of rotation may serve as an important function of grounding mathematical ideas in bodily form and they may also communicate spatial and relational concepts (Boncoddo et al., 2013) in the field of plane transformation. In general, users manipulate the screen

[1]Supported by National Council of Research and Scientific Development (CNPq).

using mainly one or two fingers and, sometimes, when working in pairs they also can share fingers or hands to manipulate some shape. Users also can interact with the device in three different ways: *with* the device itself (gyrating it in different positions etc.), and interact *on* or *from* the screen.

Touching screens constitutes a new language and therefore holds particular implications in our way of being and thinking. The handling we perform on a mobile device is a way to unveil and materialize our thoughts in a communicative act in order to favor an interaction. Interaction through mobile touchscreen basically occurs with the device recognizing and tracking the location of the user's input within the display area. This interactivity enables at least six type of manipulation (Arzarello et al. 2014): tap, double tap, long tap (hold), drag, flick, and multi-touch (rotate). In this sensorial process, motion and manipulation on screen take an important cognitive role and, in their movement into existence, in which they become objects of thought and consciousness, geometric concepts are endowed with particular determinations; they have to be actualized in sensuous multimodal and material activity (Radford, 2014).

Touchscreen constitutes another way of language manifestation and of embodied cognition. From McNeill (2002) we stress the conjunction *gestures+talk+touches+construction_on_screen+other_inscriptions* on learning process with DGEwT. Particularly, manipulation or touches on screen are not always accompanied of talk, are produced *with* the screen or *through* it and constitutes a symbolic multifaceted system (Bairral 2017).

3. Handling and performing plane transformation on DGEwT

In Brazil, even in High School or Prospective Mathematics Teacher Program, plane transformations do not appear in current official curricula. Besides alternative kinds of rotation applied by students to solve the geometric tasks, justifications to analyze students performing rotation or other plane transformations in DGEwT are the following (Bairral, Arzarello & Assis, 2017): rotation and other gyrating movements on screen are often applied due to the various alternatives of handling touchscreen devices (Kruger et al. 2005; Tang et al. 2010); rotation and other plane transformations have remained unaddressed in Brazilian geometry classrooms so far; touchscreen devices provide possibilities of gyrating movements on screen, or with the device itself, which might result in new insights on embodied cognition; and rotation and other plane transformations are concepts that involve intrinsically embodied motions.

One type of manipulation we often do with our smartphone is rotating (Bairral et al., 2017) which is accompanied by orientation (Kruger et al., 2005). Many devices constantly require objects to be rotated or transferred. We turn the screen around in order to better see some picture, video etc. We turn around our body together with the smartphone in order to share or interact with our interlocutor, for instance. Although gyration (rotation) and transfer, with or without a smartphone, are essentially bodily actions and frequent in our daily lives, they can also be analyzed from a mathematical point of view. Conceptually, in order to rotate one shape, we need to determine beforehand in each point the center of rotation, but with the use of two fingers the decision may have not been done beforehand. Arzarello, Bairral and Dané (2014) observed, in a task 1.1 (on Appendix) that didn't apply plane transformation concept, different way high school students did rotation and other gyrating movements (with two fingers in movement, one fixed finger and the other in motion etc.).

4. Methodological aspects of the study

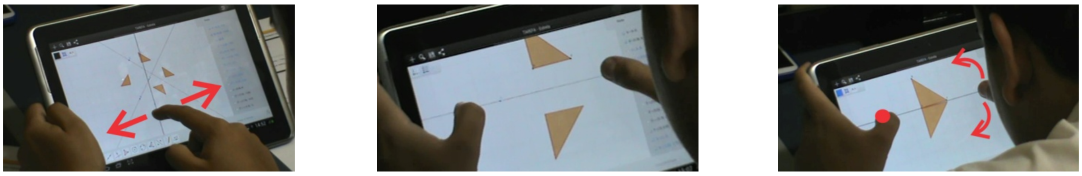
We conducted teaching experiments (from 2014 to 2016) with High School students (15-17 years old) at *Instituto de Educação Rangel Pestana* (Nova Iguaçu, Rio de Janeiro, Brazil). All of them had no previous experience with DGEwT and had no lesson concerning plane transformation. Each session was 2 hours long and in each one the students worked alone or in pairs. The analysis process was mainly based on the (1) videotapes of students working on the software, (2) written answers for each task, (3) the use of one shift in which he or she could write down and describe the function of each device icon, and (4) from recorder SCR PRO, which generate screen recording and allows the researcher to observe in details manipulations which happens "within" the device.

5. Protocol 1: Student Adriano applying plane transformation to solve task on GeoGebra

In this section we illustrate student Adriano dealing with GeoGebra (single touch) on task 4.5[2], which was mainly designed to provide the use of rotation concept[3]. He starts (12:14) constructing lines and reflecting triangles, relating with them. Moving the line (27:34) he tries to locate the triangle to become coincident, but since he has no success he decides to restart the construction.

Picture 1 - Student moving the line

12:14 | 27:34 | 28:14



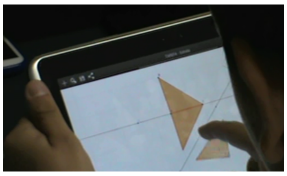
Using reflection tool and moving the line trying to adjust the reflected triangle | Restarting the construction, observing and adjusting

While observing and adjusting, it is interesting to highlight how the student keeps his left finger under some point on the line and makes the rotation of the line using his right finger. In the next figures we observe Adriano constructing lines and using reflection to move the triangles.

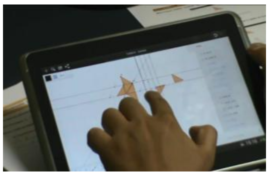
Picture 2 - Moving reflected triangles

[2]See it on Appendix.
[3]Access https://drive.google.com/file/d/0B6zQPvF8JeJcbzNsU0dMbUh2bE0/view to see the video recorded by Adriano solving task 4.5.

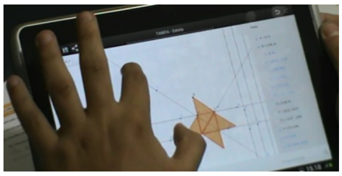
28:28-28:33 | 35:51 | 38:16



Constructing line and using reflection tool



Using reflection tool and line by two points afterword reflects the triangle

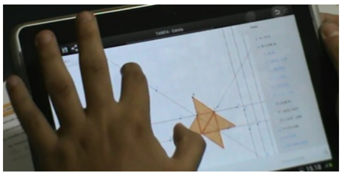


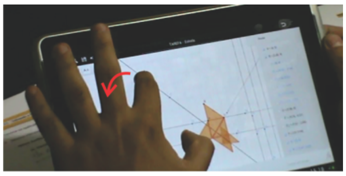
Applying rotation motion

The student constructed line (28:28) and used reflection tool (28:33) to move the triangle. Afterwards he constructed other lines and repeated the process of reflection the triangles (35:51). In the next three pictures we illustrate Adriano applying rotation motion by keeping one finger on the line. Particularly, at 38:17 he makes a rotation motion with his finger to move the triangle and complete the shape (38:18).

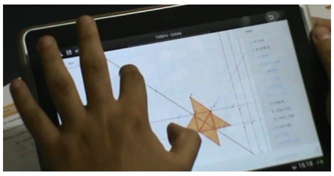
Picture 3 - Moving the triangle with the constructed line

38:16 | 38:17 | 38:18





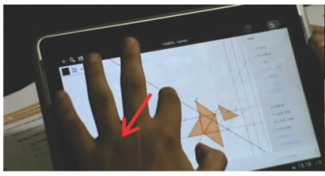
Rotate touch

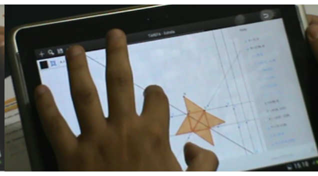


The next pictures show how Adriano was dealing with his constructions to put the triangle (38:18) in a right position according to the task.

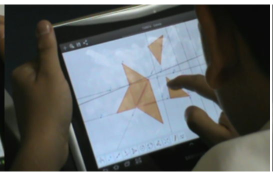
Picture 4 - Adjusting the triangle

38:19 | 38:22 | 49:48 | 51:39

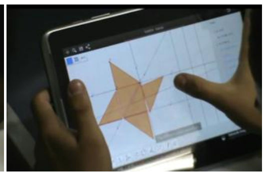




After motion realizes that triangle is inverted



Reflecting triangle



Adjusting and finishing the construction

Using the same finger that he was working with before, Adriano selects the line (38:19) and translates it in a way so that the triangles become coincident. He created one more line and reflected the triangle (49:48). Afterwards he adjusted and finished the construction according to the task statement.

6. Protocol 2: Students Adriano and Eduardo working together on Geometric Constructer

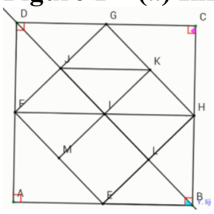
In this section we are increasing our analysis bringing data from recorder SCR PRO, in order to enrich understanding about geometric strategies emerged from solving the task

7[4] on Geometric Constructer[5]. Here we are illustrating the interaction of one pair (Eduardo/Ed and Adriano/Ad) working together on one tablet.

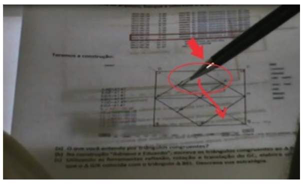
Initially, we asked the students what they understood by congruent triangles. Adriano said that they were "two figures that <u>make a new</u>[6] image". His idea was important because congruency was defined as an identity function. The sentence "make a new image" is interesting for isometry instruction due to the idea of transformation. At this moment it was agreed between students and teacher that congruent triangles were "equal triangles".

We then proceeded to ask them to choose one of the triangles in Figure 1(a) and Adriano indicated ∆ GJK. We requested they indicate congruent triangles to the one highlighted (∆ GJK). They indicated JKI, IMF, JFI e IHL. Adriano pointed out the pair DGF and CGH as a pair of congruent triangles. It is interesting to note that these two are not congruent to the triangle that was initially set out (GJK). We asked if ∆ GJK was congruent with ∆ IML and Adriano said it wasn't, because triangle IML did not exist, as the segment LM had not been drawn. Still, Eduardo said that if we closed the figure we would have a triangle, which is an indication of how geometric visualizing develops. Adriano pointed out that the triangle LME would still be formed. We asked them to join points L to B and to build a straight line that would pass through the points B and D. After this moment of probing and conceptual clarifying about congruence and possibilities of prolonging and formation of new triangles, the students were asked to elaborate a strategy so that triangle GJK would fit exactly into ∆ LEB (Figure 1b).

Figure 1 – (a) Initial construction, and (b) indicating the triangle[7]



(a)

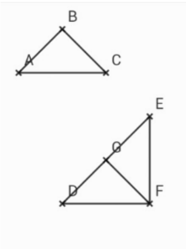


(b)

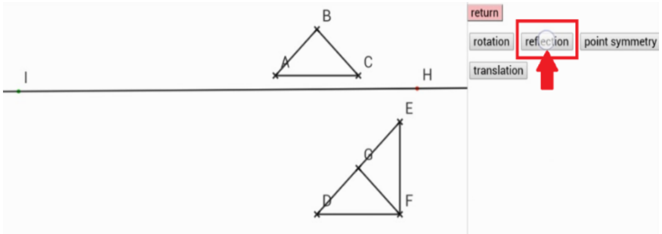
In GC, the first moment was for recognition and handling of tools: "transformation". Since triangles ABC, DGF and GFE were fixed (task restriction), the students built straight line HL, aiming to reflect some element of ∆ ABC (Figure 2b)[8].

Figure 2 – (a) Initial construction, and (b) moment of selection of the plane transformation

[4] See it on Appendix.
[5]Free DGEwT developed in Japan by Yasuyuki Iijima at Aichi University of Education.
[6] Our underlining.
[7]The video clippling from task 7 can be accessed at https://goo.gl/SXzgGE
[8] Arrow, circle, straight line colored etc. indicated in each picture or chart are researcher's strategies for tracking students' reasoning in detail by touching on screen or doing construction on software.



(a)

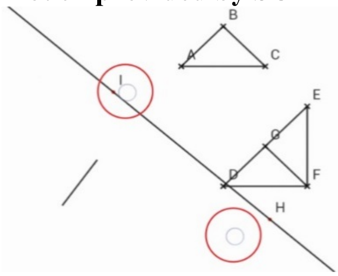


(b)

Adriano selected side AB and reflected it in relation to the straightline HI (Figure 3a). Eduardo selected point I and performed semicircular movements (without touching point H) aiming to check the position of segment AB reflected in relation to Δ DGF. Figure 3 represents moments of double touching when they were manipulating straight line HI.

Figure 3 – (a) Manipulation and (b) backstage[9] touch provided by SCR PRO

(a)



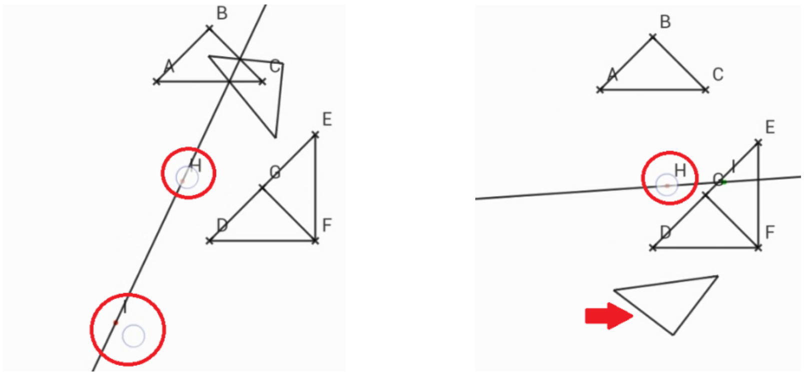
(b)

Due to the position of the straight line and the segment reflected, the manipulation enabled reflected segment AB and segment DF to be overimposed, which was enough for them to realize that the segments were of different sizes. We can see the movement performed by Adriano, when he selected point H and Eduardo moved point I simultaneously, in an attempt to make the reflected segment coincide with segment DG. Such strategy was possible to be identified only by SCR PRO (see Figure 3b). At this moment, Adriano exclaimed: “It’s ready!”.

Eduardo and Adriano manipulated the straight line IH synchronically. At moments such as these, they checked that it would be necessary to reflect the other sides of Δ ABC (BC and AC). After the process to reflect each of the sides of Δ ABC in relation to the straight line IH, Eduardo started manipulating the straight line, trying to adjust the reflected triangle to Δ GFD. We identified moments of manipulation using one or two fingers. These moments initially seemed to be random movements by one of the students, but the different positions occupied by the triangle contributed for the other student not to abandon the construction. In the process of adjusting, in the attempt to “fit” the reflected triangle into triangle DGF, we identified the composition of rotation and translation – Figure 4(a) → Figure 4(b) – of the reflected triangle, as a result of the manipulation of points I and H simultaneously.

Figure 4 – (a) composition rotation-translation, (b) adjustment

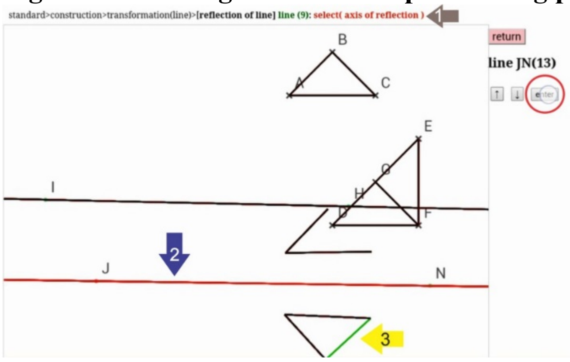
[9] Only researchers access this kind of manipulation, which we call backstage.



(a) (b)

While Eduardo manipulated and performed the rotation movement of the triangle from the selection of point H with fixed I, Adriano signaled for the need to draw one more straight line. He pointed to the reflected triangle and observed that it was inverted in relation to ∆ DGH and that, in order to “fit” it – into ∆ DGH – it would be necessary to draw one more straight line and perform one more reflection in relation to this new straight line. In Figure 5, we can see a fragment of Adriano’s process in making the second reflection. He first drew the straight line JN (arrow 2), then he realized that if he did the reflection in relation to straight line IH, he would have ∆ ABC again. The procedure that Adriano used in order to perform the reflection in relation to straight line JN followed this order: (i) the transformation, in this case, reflection; (ii) one side of the triangle indicated by arrow 3 and (iii) the axis of reflection, straight line JN (arrow 2).

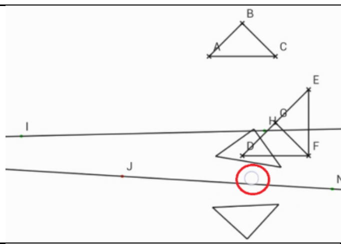
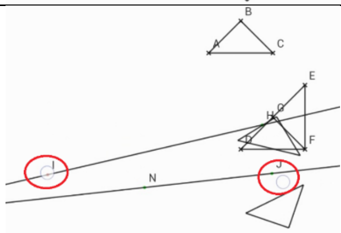
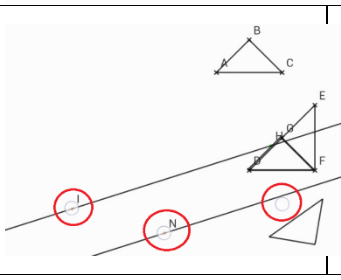
Figure 5 – Cutting of Adriano’s performing process: second reflection



GC resource multi-touch enabled the identification of some touchscreen manipulations. Students most often used one or two fingers. As we are dealing with actions performed in fractions of seconds and constructions are fast and dynamic, we present some time interval in the performing of manipulation when students aimed to adjust the second reflected triangle so that it would coincide with ∆ DFH.

Chart 1 – Composition of transformations captured by SCR PRO

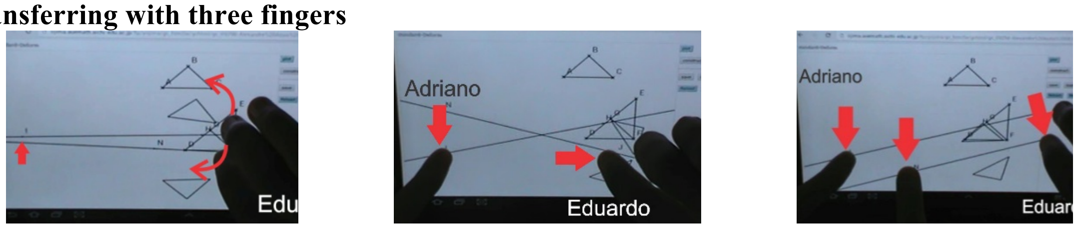
Time	Construction or manipulation captured by SCR PRO (backstage)	Description of geometric strategy and students’ reasoning (manipulating or touching on screen)

33: 56		Ad **touches and keeps his finger fixed** (shown with the red circle) on straight line JN, but he **does not get to move** the construction. He quickly selects point N and moves it. **He makes sure** that both reflected triangles were also moved.
34:46		Ad **keeps his finger fixed** on point I. Ed. Ed **touches** and keeps his finger on point J, and makes a movement that results in the rotation of the reflected triangles.
35:12		At this moment, Ad **keeps** his finger fixed on I, he touches and keeps his finger fixed on N, and together with Ed, finger on point J, **they perform a movement** that relates to a rotation and translation and they manage **to adjust** the second triangle reflected to triangle DHF

In Chart 1, at moment 35:12, SCR PRO shows the three touches performed by the students. When analyzing the recording, directed towards the students' hands movements, we can see that two touches were done by Adriano and one touch by Eduardo. Another curious fact was that we didn't get (on the recording by SCR PRO) to see point J, but then, turning to the recording done by the camera, we identified that even without visualizing point J in the manipulation area, GC responded to Eduardo's manipulations.

In Figure 6(a), Eduardo performed a circular movement whereby he manipulated point J of the straight line JN. The movement produced a rotation of the second reflected triangle. The manipulation performed by Adriano, illustrated in Figure 6(b), created a situation where the students realized they could handle more than one element at one single time, and in order to do the adjustment, as they initially intended, Adriano keeps his finger on point N, and together, going through a composition between transferring and rotation, they perform the adjustment of the reflected triangle, with Eduardo manipulating point J and Adriano points I and N. The moment they performed three simultaneous taps, as can be seen in Figure 6(c), is especially noteworthy.

Figure 6 – (a) Manipulation with one finger (Ed), (b) rotation with two fingers, (c) rotation and transferring with three fingers



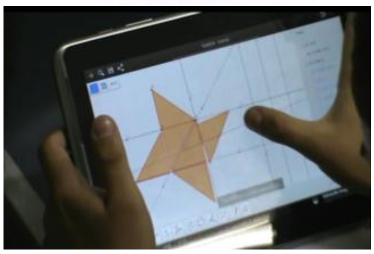
(a) (b) (c)

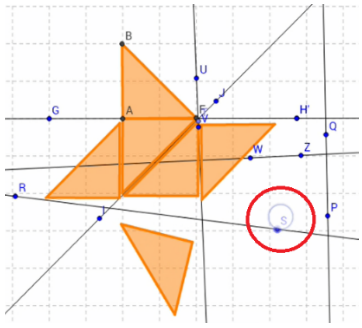
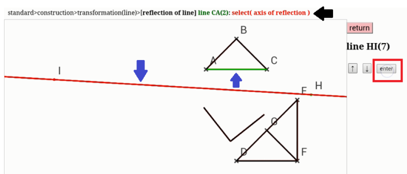
During the choosing of icons for the construction, or for the touchscreen performing, actions were accompanied with reasoning that indicated what was being performed, hence providing clues as to how the manipulation of the screen was putting together the students' discoveries.

7. Summarizing results

Conceptually, in order to rotate one shape we need to determine before in each point (the center of rotation) and with the use of two fingers the decision could have not been done beforehand. This type of action was not explicit for students exploring task 1.1. From protocol 1 we identified that students applied rotation and reflection concepts naturally, sometimes even doing composition between them. We became intrigued and kept investigating (protocol 2) new conceptual aspects for the way we deal with rotation and other gyrating movements (with two fingers in movement, one fixed finger and the other in motion etc.) as observed in the three tasks illustrated in this paper and summarized in the following chart.

Chart 2: Example of students' performing plane transformation in some of the proposed tasks

	Device	Protocol discussed in this paper	Picture	Summarizing geometric strategies
Task 1.1	Geometric Constructer (multi-touch)			Student keeps one finger fixed (from the left), moves the middle and observes what happens. Student rotates freely using two fingers but with one fixed finger
Task 2.2	GeoGebra (single touch)	1	 	Using pencil and paper students identified a square on the center of the red picture. Afterwards they started work on the device. Task had no restricted conditions. Inspired by square students added the squared grid measured each size of the square and constructed the triangles. After doing that they worked out to adjust the position from each square but they did not applied some plane transformation. Their strategy was based on measure and square grid provided by GeoGebra.

Task 4.5	GeoGebra (single touch)			As show in Appendix for solving this task students had to use some icon. From the original triangle, he used the icons  and  to reflects it. After <u>mirroring the pictures several times</u>, he was moving and fixing the figure to the <u>correct position</u>. Student strategy was mainly based on those two icons, named by him as "mirror" and "line of mirror", respectively.
Task 7	Geometric Constructer (multi-touch)	2		Like task 4.5 this one also had some conditions in it statement. Inspired on strategy created in task 4.5 (<u>line of mirror</u>, in red) for solve task 4.5 students started applying axial symmetry on triangle ABC based on "line of mirror". Afterwards based on such line they recursively applied translation or rotation for adjust their construction or shapes to solve the task.

When solving task 2.2, which involved the concept of rotation and using a device with a single touch, we observed that students used their fingers – no more than two (Tang et al., 2010) – in a similar way to what students did when dealing with software Geometric Constructer in task 1.1 which did not apply the referred concept. Although the task 1.1 had been designed (without a specific geometric concept) for free exploration and to know the software, the students made a lot of interesting gyrating movements. After observing such way of manipulation we elaborated a set of tasks, for which students have to apply the concept of rotation and other plane transformation. From analysis illustrated on protocol 2 we observed (as show in Figure 4) reasoning based on mirroring ideas in which students' used axial symmetry as scaffolding concept during their interaction to solve the task 7.

The iterative task design was mainly based on two strategies: task that generated new (or reformulated) task (for example, task 2.2 became 4.5) and students' answer that inspires new task (for instance, task 7[10] was elaborated according answer from task 6).

[10]See it in Appendix.

At the moment of getting familiarized with Task 6, students made a construction with plenty of details (as shown on Figure 2 and chart 1) and geometric properties, which made us realize that they could, from their own construction, elaborate a new task that would permit observing and analyzing possible forms to perform rotations, once the Geometric Constructer allows the manipulation of construction elements with multiple simultaneous touches.

8. Final remarks

The type of task has an important role in the growth of the mathematical thinking. For researchers it also bears influence on the findings. The way in how a multi-touch-screen is used allows alterations on the task design in a substantial way. The kind of task needs to be strongly interconnected with the choice of the device and its features and artifacts mediators. For instance, see differences between task 2.2 and 4.5. By taking device features and performances into account, we conclude that teachers need to be aware of the singularity of each proposing tasks that aim to trigger the students' intrinsic motivation to work into mathematics activities that enhance findings, reflections, and the development of mathematical thinking in its various aspects (Bairral et al., 2015).

In terms of promoting new ways to discover and to think mathematically, it doesn't make sense to propose, for instance, task 2.2 using only pencil and paper. As we have shown in the analytical section, task design with touchscreen device allowed students – without previous lessons concerning rotation, symmetry or translation – to apply those concepts naturally, sometimes isolated, or even doing composition between them. The possibility of to make different constructions, to do simultaneous movements and adjusting by touch on screen seems to be a powerful resource for changing tasks as well as the nature of the geometric understanding concerning plane transformations using DGEwT.

Usually in Brazil, plane transformations are conceptually mapped in the following sequence: reflection/axial symmetry, rotation and translation. The composition of plane transformations is underexplored in geometry lessons. In our teaching experiments students had no previous lessons with plane transformation, but they applied intuitively reflection, symmetry, rotation or translation concepts, isolated or mixed (Assis 2016).

Due to nature of the device and freedom on movements and performances we suppose that is difficult to track some conventional sequence or linear strategy on students reasoning. Also, plane transformations are concepts that involve intrinsically embodied motions and became usual in our day life when we perform some manipulation in our touch device. In current analyses, we are checking whether the students use one and the same sequence in their reasoning, or if their strategies emerge naturally and without the traditional linearity taught in Brazilian schools (reflection/symmetry → rotation → translation). We have to go further in such analysis and screen recordings are helping us for deep understanding plane transformation learning process within DGEwT.

The four tasks described in this paper, among others created and implemented by Assis (2016), were redesigned by teachers and researchers within our research group. This reflective process allows us to improve our learning in different ways, for instance, in geometrical (understanding and analyzing ways for doing composition of plane transformation), technological (knowing better devices' performances) and pedagogical aspects (elaborating different tasks taken into account device features), and to understand better how to conduct design research based on touchscreen devices. These theoretical results are also essential both for designing suitable teaching situations and for elaborating observation protocols for investigating students' behaviors when they are exposed to mathematics problem solving based on the actions with a multi-touch device.

9. References

Arzarello, F., Bairral, M., & Dané, C. (2014). Moving from dragging to touchscreen: geometrical learning with geometric dynamic software. *Teaching Mathematics and its Applications* 33(1): 39-51.doi: 10.1093/teamat/hru002

Assis, A. R. de (2016). *High school students working in GeoGebra and in Geometric Constructer*: *Hands and rotations in touchscreen.* Master Thesis in Education. Federal Rural University of Rio de Janeiro: Institute of Education.

Bairral, M. A. (2017). As Manipulações em Tela Compondo a Dimensão Corporificada da Cognição Matemática. *JIEEM, 10*(2), 104-111.

Bairral, M., Arzarello, F., & Assis, A. (2017). Domains of manipulation in touchscreen devices and some didactic, cognitive and epistemological implications for improving geometric thinking. In G. Aldon, F. Hitt, L. Bazzini, & U. Gellert (Eds.), *Mathematics and technology: a CIEAEM source book* (pp. 113-142): Springer.

Bairral, M., & Assis, A. (2017). Touchscreen devices and task design to improve plane transformation in high school classroom. *Quaderni di Ricerca in Didattica: Matematica* 27 (Supplemento n. 2), 135-139.

Bairral, M. A., Arzarello, F., & Assis, A. (2015). High School students rotating shapes in GeoGebra with touchscreen.*Quaderni di Ricerca in Didattica: Matematica* 25 (suplemento 2), 103-108.

Bairral, M., Assis, A. R., & Silva, B. C. da. (2015). *Mãos em ação em dispositivos touchscreen naeducação matemática*. Seropédica: Edur.

Boncoddo, R., Williams, C., Pier, E., Walkington, C., Alibali, M., Nathan, M., Dogan, M.F. &Waala, J. (2013). Gesture as a Window to Justification and Proof. *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. M. C. S. Martinez, A. Chicago, IL, University of Illinois at Chicago: 229-236.

Damásio, A. R. (2010). *O livro da consciência: A construção do cérebroconsciente*. Porto: Temas e Debates.

Kruger, R., Carpendale, S., Scott, S.D., Tang, A. (2005). Fluid Integration of Rotation and Translation. Paper presented at the Proceedings of the ACM Conference on Human Factors in Computing Systems (CHI)'05, April 2-7, 2005, Portland, Oregon, USA, pp. 601-610.

McNeill, D. (2002). Gesture and Language Dialectic. *Acta Linguistica Hafniesia*, 1-25.

Radford, L. (2014). Towards an embodied, cultural, and material conception of mathematics cognition. *ZDM – The international journal on Mathematics Education, 46*(3), 349-361.doi:10.1007/s11858-014-0591-1

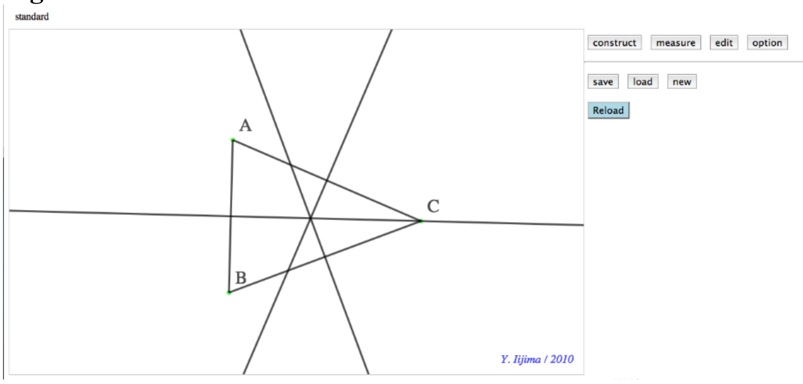
Tang, A., Pahud, M., Carpendale, S., & Buxton, B. (2010). *VisTACO: Visualizing Tabletop Collaboration.* Paper presented at International Conference on Interactive Tabletops and Surfaces (ITS '10), Saarbrücken, Germany.

Appendix

Task 1.1: *For introduction and familiarization with Geometric Constructer device (30 minute)*[11]

Use the software commands (construct, measure, etc.) to understand their functions, them draw the triangle using the commands on the iPad; write your remarks. Before exploring the software write down two observations:

Figure: Screen from GC



Task 2.2 (design 1): *Stair task*

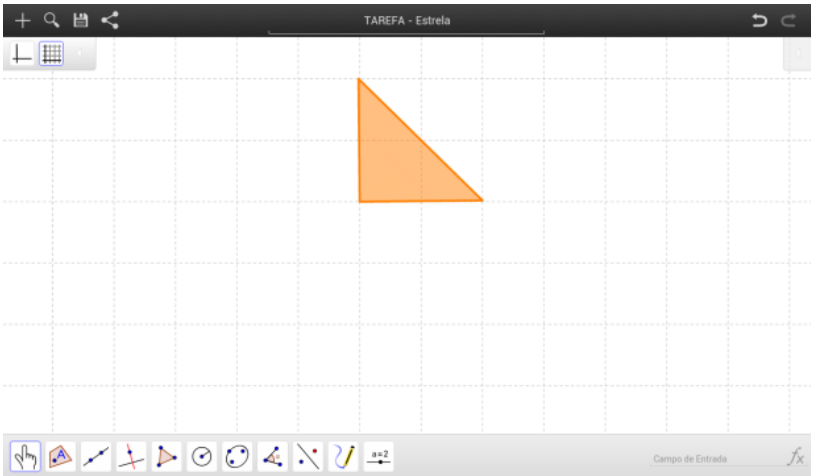
Using only triangle rectangle and isosceles construct the following picture.

Now, write to a friend and tell him or her how you constructed the picture.

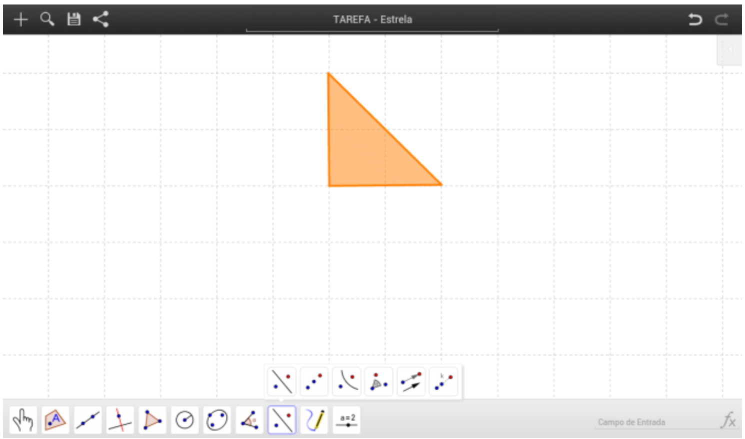
Task 4.5 (design 2 from task 2.2): *Stair task*[12]

Open the file "Stair task". Only the following triangle will appear:

[11] Links where to find the software and this activity:
a) with PC http://www.auemath.aichi-edu.ac.jp/teacher/iijima/GChtml5/GChtml/server_e/gc_00026-test.htm
b) with I-pad:2012/10/10 16:39 482434 gc_00026-test.htm
[12] This version restricts the use of icon.



Selecting the tool  will open a bar with 6 options:



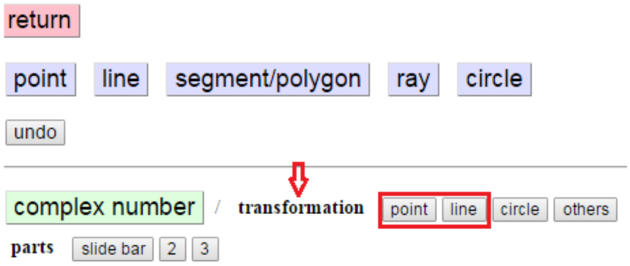
Elaborate a strategy to construct the following picture using only the tools 

***Task 7:** Moving triangles*

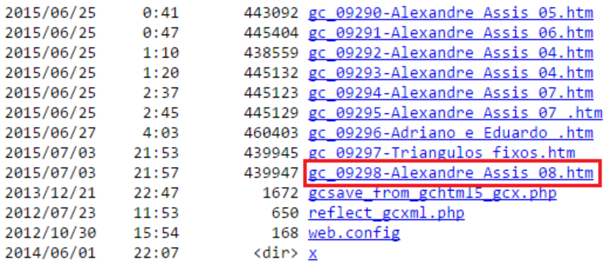
We are going to explore some of the GC tools. Build a triangle, any triangle whatsoever, and a straight line. Now we are going to enable some features. Carry out, in order, the commands represented in the following sequence:

option → mode → expert → return → construct

After carrying out the commands, we are going to have this figure represented on the screen:

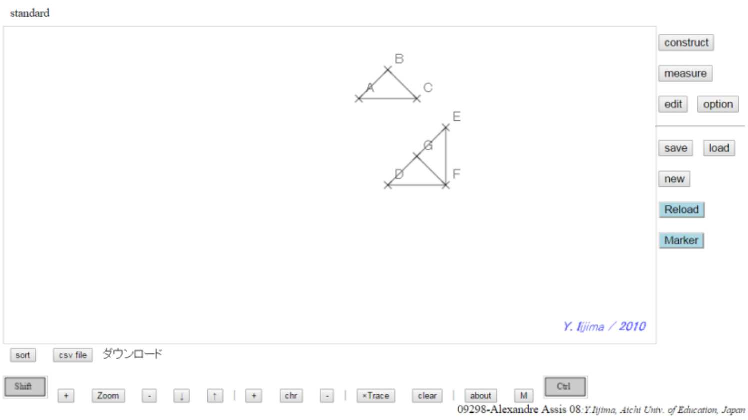


We are going to explore the transformations rotation, reflection and translation. Select the options “point” or “line” and, as the built triangle and line using some of these transformations – check what happens in every one of their actions. After exploring, open the construction “Alexandre Assis 08”.



2015/06/25	0:41	443092	gc_09290-Alexandre Assis 05.htm
2015/06/25	0:47	445404	gc_09291-Alexandre Assis 06.htm
2015/06/25	1:10	438559	gc_09292-Alexandre Assis 04.htm
2015/06/25	1:20	445132	gc_09293-Alexandre Assis 04.htm
2015/06/25	2:37	445123	gc_09294-Alexandre Assis 07.htm
2015/06/25	2:45	445129	gc_09295-Alexandre Assis 07 .htm
2015/06/27	4:03	460403	gc_09296-Adriano e Eduardo .htm
2015/07/03	21:53	439945	gc_09297-Triangulos fixos.htm
2015/07/03	21:57	439947	gc_09298-Alexandre Assis 08.htm
2013/12/21	22:47	1672	gcsave_from_gchtml5_gcx.php
2012/07/23	11:53	650	reflect_gcxml.php
2012/10/30	15:54	168	web.config
2014/06/01	22:07	<dir>	x

You will find the following figures on screen:



Develop a strategy so that the Δ ABC coincides with the Δ DFG. Describe your strategy.