ACTIONS IN SEARCH OF A MORE INCLUSIVE MATHEMATICAL EDUCATION

AÇÕES EM BUSCA DE UM ENSINO DE MATEMÁTICA MAIS INCLUSIVO

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ABSTRACT

In this article, we present some discussions about the school inclusion of people with disabilities in Brazil. The Brazilian laws signal a step in terms of legislation in favor of inclusive education, but they are not enough without the actions that will remove them from the paper. We bring the actions carried out by a Geometry Teaching Lab. These actions try to attend, in the training of Mathematics teachers, the current legislation and the demand of students included in common schools, especially those with visual impairment, promoting also Inclusive Mathematics Education and equity in Mathematics teaching, mainly Geometry, in the that all students can participate with the minimization of visual barriers through low cost didactic resources (concrete and virtual).

Keywords: Adapted didactic material; Axioms of Euclid; Mathematics Laboratory; People with visual impairment.

RESUMO

Neste artigo, apresentamos algumas discussões sobre a inclusão escolar de pessoas com deficiência no Brasil. As leis brasileiras sinalizam um passo em termos de legislação a favor da educação inclusiva, mas não são suficientes sem as ações que as removerão do papel. Trazemos as ações realizadas pelo Laboratório de Ensino de Geometria da Universidade Federal Fluminense. Essas ações tentam participar, no treinamento de professores de Matemática, da legislação atual e da demanda de alunos incluídos nas escolas comuns, especialmente aqueles com deficiência visual, promovendo também a Educação Matemática Inclusiva e a equidade no ensino de Matemática, principalmente a Geometria, naquilo os alunos podem participar com a minimização de barreiras visuais através de recursos didáticos de baixo custo (concreto e virtual).

Palavras-chave: Material didático adaptado; Axiomas de Euclides; Laboratório de Matemática; Pessoas com deficiência visual.

1. Introduction

The discussions related to an inclusive teaching are not recent. In Brazil, the Declaration of Salamanca, from 1994, can be considered as a starting point and a matter for discussion, when the country establishes the commitment of "Education for All", with the compromise to transform the educational system in order to receive everyone, indiscriminately, with quality and equality of conditions (Unesco, 1994).

After this initiative, governmental policies were created with recommendations such as compulsory enrollment of students with disabilities, pervasive developmental disorder (PDD) and high abilities or giftedness and the training of teachers and the adaptation of the institutions in all governmental areas.

Among the most recent laws is the National Plan of Education (PNE, in Portuguese, 2014-2024) which in its Objective 4 reinforces the universalization of the access to the basic education and the specialized educational support to the target public of Special Education. It should preferably take place in regular schools, with the guarantee of the inclusive educational system, multifunctional resources rooms, classrooms, specialized schools or services, public or partners. Hence, it also recommends the training of graduates and undergraduates for the teaching and promotion of the Inclusive Education (Brazil, 2014a). And the law number 13,146, from 6th of July, 2015, which institutes the Brazilian Law of Inclusion of the Person with Disability (Statute of the Person with Disability) in its article 28 delegates to public authorities the duties of assuring, creating, developing, implementing, encouraging, accompanying and evaluating:

> I – inclusive educational system in all levels and modalities as well as continuous learning throughout life; II - improvement of the educational systems, aiming at guaranteeing conditions for access, continuity, participation and apprenticeship by offering services and resources of accessibility in order to eliminate barriers and promote a full inclusion; III – pedagogical project that makes possible and official a specialized educational support, and also other services and reasonable adaptations in order to meet the needs of students with disabilities and ensure their full access to the school curriculum in equal conditions, promoting the achievement and the practice of the students' autonomy; (Brazil, 2015a, p. 4)

These Brazilian laws indicate a step forward in terms of legislation in favor of an inclusive education, but they are not enough without the actions to take them out of the paper.

Educational data are very worrying. In the School Census report it is possible to observe that students enrolled in the modality Special Education are concentrated in public schools. In 2008, there were 205,475 students in special classrooms and exclusive schools, and 23,137 in regular schools (included students). In 2014, these figures were 136,302 and 43,393, respectively, representing a variation of -33.7% and 87.5%. In regular public schools, in 2008, there were 114,449 students in special classrooms and exclusive schools and 352,638 in regular schools (included students). In 2014, these figures were 51,745 and 655,375, respectively, representing a variation of -54.85% and 85.8% (Brazil, 2015b). It is still possible to observe the tendency of a reduction in the number of students enrolled in specialized (or exclusive) schools and an increasing number of these students with disabilities included in regular classrooms.

Considering these facts, in this paper we present the actions developed by the Geometry Teaching Lab (LEG, in Portuguese), situated in Niteroi (city in the metropolitan region of Rio de Janeiro, Brazil) and in the Institute of Math and Statistics of the Federal Fluminense University (UFF, in Portuguese). These actions try to comply with the current legislation and the demands of the included students in regular schools. They also promote Inclusive Mathematical Education and the equity in the Math teaching, especially in Geometry, allowing all students to participate with the reduction of visual barriers via pedagogical resources either overcoming the difficulty of creating mental objects or being unable to see.

2. The Geometry Teaching Lab facing the Brazilian educational reality

In search of an inclusive school, the actions of the Geometry Teaching Lab have been dedicated, since 2008, to the training of undergraduates in Math in order to adequate their professional future to the needs of inclusive education. Therefore, there is a project entitled “Seeing with the hands” in which special pedagogical resources and adapted activities are developed, dedicated to visually impaired students.

This and all the other projects developed by the Lab aim at democratizing the knowledge developed at the university. They take into account the fact that the majority of Brazilian basic schools teachers have low acquisitive power. They also consider the social reality of students with or without disabilities who belong to lower social classes.

Facing this reality and searching for both social and educational equality, the concrete teaching resources created at the Lab are traditionally built from low cost scrap materials easily bought in the market. For this special project, aimed at visually impaired students, the teaching resources of the Lab collection have been adapted with the use of appropriate materials to tactile perception with various textures. Different types of paper, cardboard and rubberized flat materials of several thicknesses and textures were used, among other materials such as cellulose acetates, wood shavings, straws and varied sewing threads.

At the Geometry Teaching Lab there are activities developed or adapted from toys and teaching materials, available in the market or described in textbooks, such as puzzles, logical blocks, golden material, various kinds of tangrams etc. (Kaleff A. M., 2010; 2016a; 2016b).

3. The vocation of the Geometry Teaching Lab and the Interactive Museum of the Lab (LEGI, in Portuguese): the actions of the Lab for the formation of the collection of teaching resources and the theoretical principles

The basic school students are indirectly benefited by the teaching resources developed at the Lab. These resources are meant to be used in the classrooms, teaching labs and also exhibitions and science fairs with “Interactive Museum” types of activities. These exhibitions are interactive in the sense that the visitor is led to participate actively and to manipulate modeling materials of mathematical situations displayed in theme “islands” separated by different Math contents. By the end of 2017, there are about 80 manual resources related to Mathematical contents and properties available at the Lab museum collection.

The interactive exhibitions called LEGI Museum have as objective the dissemination and popularization of Math, and its displays are shown in the different venues of the

Federal Fluminense University or in other places (schools, universities etc) when there are events and gatherings related to teachers training and the teaching of Math. (Kaleff A. M., 2012; 2016b).

It is important to emphasize that the name LEGI of the Museum represents the Lab vocation in its broad search for integration in the teachers training process, with focus on a meaningful Math teaching, through the interactivity of the students (with or without disabilities) with manual resources. Besides, the search for inclusion and social equity in the undergraduates training process is influenced by the experiences of spreading the Lab activities to different and remote areas. It may happen as the LEGI Museum allows members of its team to travel to different interior regions of Brazil, beyond the walls of the University which holds the project, in Niteroi. It is possible due to the disciplines of a distance learning specialization course for Math teachers offered by the Federal Fluminense University. Therefore, all these actions of the Geometry Teaching Lab (LEG in Portuguese) are expected to have the vowel "I" added, meaning integration, interactivity, interior, itinerant and inclusion – LEGI.

The manual teaching resources of the Lab collection are created by the teachers of the Lab and the students during the two mandatory disciplines of the evening Math degree course at the Federal Fluminense University. These lessons happen at the Lab.During these courses, as we will present later in this paper, the future teachers are prepared according to theoretical principles under the perspective of a Mathematical Education which aims at an active learning process, creative and meaningful for the student. (Kaleff A. M., 2016a; 2016b).

We point out that, as Math educators, we intend to prepare teachers who are not mere agents of transmission of Math contents, but mediators and motivators of the apprenticeship via a more adequate teaching and learning process considering both the student and the surrounding reality. Learning which may provide security to the person and also his/her own knowledge and life experiences to confront his/her needs for life (daily and professional), allowing more autonomous and respectful individuals concerning their peer knowledge/experiences.

Initially, it is important to observe how effective interaction takes place with each manual teaching resources (modelers such as various games, flat or three-dimensional puzzles, modeler devices of geometric shapes and surfaces, different abacus etc) that is part of the Lab collection. This interaction is possible with an *Activities Book* which accompanies each teaching resource. This book is made of cardboard paper covered, on one side, by a sheet of paper and on the other side, with appropriate paper for Braille printing. In the sheet of paper the activity text is printed in enlarged font of at least 18 points and, on the other side, in Braille. The objective of this book with these two forms of printing is to allow the students who can see or who are visually impaired to accomplish the task proposed autonomously, with the handling of the resources, without the need of help from people who can see. At the end of this *Activities Book* there is a *Teachers' Guide*, in which the objectives of the activities are described, with adequate age levels for each activity, a brief description of the materials used and the Mathematical pre-requisites needed for the accomplishment of the tasks.

It is also relevant to notice that, in the *Activities Book*, the tasks proposed, related to each manual teaching resource (concrete or virtual) try to enable the teacher to act as a real tutor of the student in search of knowledge. In other words, the teacher can encourage and guide the student to the discovery and to the development of the meanings of the Mathematical concepts. The activities allow the student to interact with the resource (game, device, artifact etc), to become conscious of the Mathematical properties modeled by them and to discover the graphic representation (layouts of drawings and graphics) or linguistics (via symbols and signs) which represent the concept. In this way, in the Lab, departing from a manual teaching resource, the tasks aim at leading the students to their (semiotic) Mathematical representations.

Some of these activities are also available online[1] with instructions for the teachers – those who have visited the exhibitions and are closer to the University or those who are far away; they enable teachers to create manual teaching resources and do the necessary adaptations for their students.

It is important to emphasize that the production of the strategies about how to conduct the tasks presented by the *Activities Book* regarding the teachers as mediators in their work with the students and also their interaction with the teaching resources agrees with the ideas of the Mathematical educators Bartolini Bussi, Alessandra Mariotti and also Raymond Duval. At the Lab, the teaching resource is considered as a toll for semiotic mediation when used by the teacher to intervene intentionally in the learning of a content through symbols and signs. (Bussi & Mariotti, 2008; Machado, 2003)

We would like to mention that, as the Lab promotes actions aiming at the inclusive school of the 21st century, the theoretical concepts of these actions are based in four principles. The first consists of guiding principles of the teaching of School Geometry in this century and for the inclusion, according to the *Catania Questionnaire* (Mammana & Villani, 1998); and with the *National Curricular Parameters* (PCN, in Portuguese) for the teaching of Math for basic and high school, as well as the *Curricular Adaptations* proposed by the Department of Special Education (Brazil, 1998a; 1998b; 1999; 2006). The second considers the ability of visualization and its relation with the development of geometric thinking of people who can see, according to Van *Hiele's Model* (Van Hiele, 1986). The third principle is included in the actions dedicated to the understanding of the role of multiple languages present in the graphic representations used in Math, considered by Raymond Duvall (Machado, 2003). Finally, in the articles published by the Magazine Benjamin Constant of the Institute Benjamin Constant (IBC) the main sources of reference used for the teaching and learning of the visually impaired students are found.

It is relevant to cite that since 2014, with the creation of the Brazilian governmental program *National Pact for Literacy in the Correct Age (PNAIC, in Portuguese)*, in which some guiding documents for basic school were presented, concerning Math literacy, they were integrated to the Geometry Teaching Lab studies. This governmental program has been implemented in public schools through teachers training courses for teachers of the initial years in municipal schools and also by the dissemination of the PNAIC booklets. These booklets present the guidelines that structure the Math curriculum for children aged between 6 to 8 years, as well as a variety of methodological resources, including ways "of making the most of the context and problem situations, with approaches that help students to learn Mathematical facts, concepts and procedures which are useful to solve real problems and to develop logical thinking" (Brazil, 2014b, p. 5).

[1] Portal of the Project Digital Contents for the Teaching and Learning of Math and Statistics – CDME/UFF, in Portuguese (www.uff.br/cdme) and the Portal of the Teacher (http://portal.mec.gov.br/portal-do-professor).

Certainly, after every new educational governmental guideline concerning the Schools or the Math teachers training courses, the guiding principles of the Lab actions are revised and reorganized.

4. The teachers actions towards the concrete and virtual manual teaching resources

Brazilian Mathematical educators point to the fact that the use of manual teaching resources (concrete or virtual) by the teachers is usually based on two reasons: themotivator aspect and the expectation that the resources may facilitate and ease the students' learning difficulties (Miorim, 1990; Passos, 2006). Both justifications are legitimate, considering that the teachers, in most cases, is solitary to face the lack of interest of students concerning Math. Also, teachers are alone to deal with the issues related to overcrowded and uncomfortable classrooms, lack of valorization and social stimulus, lack of appropriate training regarding the current scientific and technological moment for appropriate inclusion, among other issues.

The reasons presented legitimate the use – or not – of a resource and also any kind of teaching strategy which involves this resource. For example, the simple presentation of manual materials in classroom without a real contribution for a meaningful learning of the Mathematical concepts and relations is useless. As we have pointed out, it is not only the specific features of the resources (color, texture, ludic potential etc) that must be taken into consideration. In other words, it is not only the format of the modelling device of a Mathematical situation (noticeable by human senses) or the ludic potential of the game (for the student's involvement with it) that must be considered when a teacher chooses the activities. However, this choice must be based on the functionality of the teaching resource regarding the motivator element of the student's action and also an active mediator in the development of the students' thinking process in search of a meaningful and creative teaching and learning process (Kaleff A. M., 2016a; 2016b). In order to exemplify what is done at the Lab, we present a set of teaching experiments idealized and made there. In other words, we will show the manual resources and the activities. In this study, we try to create resources of a very low cost which help the process of teaching the introductory elements of Euclidean Geometry for elementary and high school students. The activities were developed with students who can see and with visually impaired students at the School Pedro II, in Rio de Janeiro, Brazil.

The theme chosen was "Introduction to the five axioms of Euclidean Geometry", based on our study for students who can see entitled *"If Euclides folded papers… an introduction to the five first axioms"* (Kaleff A. M., 2016a). The studies of the programmatic contents were based on the axiomatic theory developed by David Hilbert and form the theoretical framework for the axioms of Euclidean Geometry (Barbosa, 2004).

The choice of the theme also considers the concern with the results of a research done in 2013, as part of a discipline about Geometry teaching points, belonging to a distance learning specialization course for teachers of Math, offered by the Federal Fluminense University. There were 136 teachers involved in the research, living in three of the largest Brazilian states. The teachers answered an extensive questionnaire with 30 questions. Analyzing the data, the researchers collected information about the teachers training process to work with Geometry and with inclusive education. This research points to the fact that the teachers probably have difficulties to deal with students who come from the initial basic school years in their Geometry classes. We were shocked by the answers to the question: "*When, in your training process, did you get in touch with the contents about Euclidean Geometry?"* Surprisingly, around 17% of the participants answered that they only got to know these contents in the disciplines of the specialization course. On the other hand, only 64% of the participants said they hadthese contents during their graduation in Math. About 6% said they got these Euclidean contents in other graduation courses, and 1% said they learned the contents teaching, in practical life.

We also observed that these teachers had very little or almost no contact with issues related to inclusion in their training courses, as we noticed another worrying reality: less than 45% of the teachers had some kind of specific discipline about Inclusive Education in their training courses – graduate or postgraduate courses. And only about 20% of the teachers had special training in Braille, Language of Signs (Libras, in Portuguese) etc, other than their training processes (Kaleff, Rosa, & Dornas, 2014).

We can add that about 16% of the teachers who participated in this study admit having had their first contact with Inclusive Education during the discipline of the specialization course here considered. They also point out how important this discipline was to them. We mention the report of a teacher with a degree in Math, who had graduated around two years before the research. She can see and had never taught disabled students. She said: *"During the discipline I could improve my way to approach the inclusion of students with special needs in the classroom, and also how to evaluate the students using other means, during the teaching and learning process. I have learned that each student has his/her own way to learn, and we, as teachers, have to know how to identify this through our daily classroom experiences, knowing the students and their levels of learning*"

A second opinion, which refers to the same discipline, from another Math teacher who can see, graduated approximately two years before the research and had already taught visually impaired students*: "Besides the academic support offered during this discipline, I need to emphasize the importance of the resources presented that we can use with our visually impaired students. These resources give us a good structure to promote in the classroom social inclusion, working with our students the sense of citizenship".*

Another teacher who is able to see, when asked if he had changed his practice as a student during the mentioned discipline answered that *"In fact, the changes that happened were, basically, in the way the practical activities created by the Lab and the interactive collection were presented, even though they are virtual; the awareness of the importance given to the visual stimuli, the development of the manual abilities and the precious lessons regarding the teaching and learning process of visually impaired students."*

The data above presented only reinforce the importance of actions that come out of the paper to supply not only the students' education, but also the teachers'.

5. The Educational Experiments: "Introduction to the five Euclidean axioms"

Initially, before we present the activities designed for the students, we need to bear in mind that there is no doubt that *"The Elements*", written by the Greek Euclides, in the

3rd century b. C., represents the most important contribution of the antiquity for the development of the Science. Euclidean Geometry became the way that the researchers, interested in understanding the natural phenomena and the environment, describe and present the characteristics of our physical universe.

In the first school years, it is important that the teacher emphasizes to the students that Geometric shapes or patterns may be used as the most elementary patterns for different types of phenomena in daily life and our surrounding world.

In the last decades, it has been said that Geometry has fundamental importance in the students' development. It helps them to solve daily life problems, in a better use of tridimensional space, the development of the abilities of visual and tactile perception, the visualization and the establishment of connections between Math and the other areas of knowledge. In fact, according to the National Curricular Parameters:

> […] the geometric concepts are an important part of the Math curriculum as, through them, the student develops a special type of thinking which allows him/her to understand, describe and represent, in an organized way, the world where he/she lives (Brazil, 1998a)

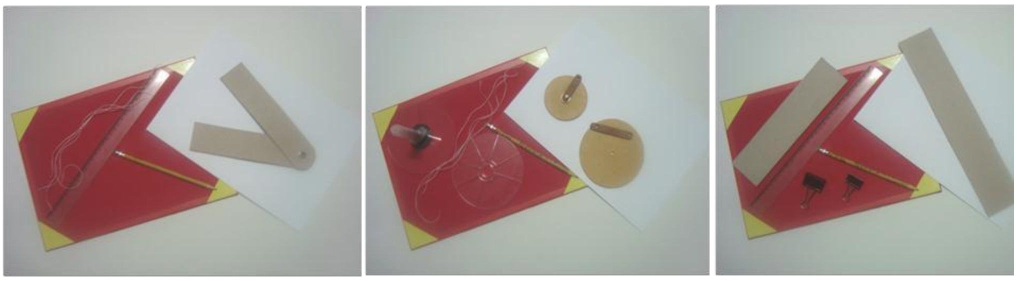
In this direction, reflecting about how to teach the introductory axioms of Euclidean Geometry to very young students in a fun and meaningful way, also considering social and educational equity – as everything that is created at the Lab – the tools for the educational experiments here shown were made of low cost materials. They were also adapted with high relief, aiming at accessibility and inclusion of visually impaired students. We cannot ignore the fact that those who are not disabled may also benefit from these activities. We observed that these students aged around 11 have made the activities easily while those disabled students can do them at the age of 15.

The objective of the activities is to lead the student to build the most elementary Euclidean flat concepts. In other words, the point, straight line, plan, ray, line segment, angles, parallel lines, perpendicular lines, pencil of lines, circumference and direction.

The activities were designed to be done individually, without the teacher's help. However, the students may make the most of them interacting with their peers. In order to offer the students this interaction, we advise that the students should be distributed in the classroom in groups of four, performing the activities individually. We point out that we observed the importance of the exchange of ideas among the students of the groups, as this helps every student to advance in his/her logical thinking.

The concrete manual teaching resources used in these experiments are a *support board* made of cardboard and coated with rubbery plastic, like this: The support board was built with 4mm cardboard, size 31cm x 23 cm. It was coated with rubbery plastic of 0.3 cm thickness, made of Ethylene Vinyl Acetate (EVA) with four angle brackets made of this material. With them the student can fix an A4 sheet of paper on the board (Kaleff A. M., 2016b). Besides, in the experiments, we used special grammage paper – 180g, used for writing in Braille; two rulers overlapping, joined in one of the corners, made of cardboard; two plastic discs with different perforations; a siryinge and a plastic disc with various perforations, paper clips and also cardboard charts coated with Braille paper, in which geometric shapes were drawn in high relief.

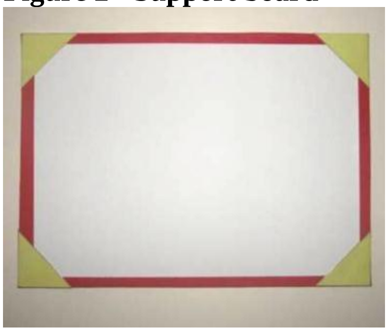
Figure 1 - Overview of the resources



Source: Geometry Teaching Lab collection

The support board is used by students who can see or are visually impaired. They use a sheet of paper and a pen to make the drawings. The blind students can use an appropriate paper for writing in Braille, with 120 g of grammage or even a sheet of waxed paper 180g to draw figures in high relief with the tip of a pen which is not too fine.

Figure 2 - Support board



Source: Geometry Teaching Lab collection

5.1. The Activities Books

The activities related to the teaching resources above described are divided in three books which intend to guide the visually impaired student, using the sense of touch, to visualize the geometric shapes, analyze their determining features and, finally, learn their meaning or define them.

Figure 3 - Overview of the Activities Book

Source: Geometry Teaching Lab collection

In the Activities Book 1, there is the introduction of the two first activities with the presentation of the basic concepts of Geometry, such as *point*, *straight line and plane.* The visually impaired student is guided to associate the points in the writing Braille system with the set of points in the plane. While in a thin, well-stretched thread, suggests a representation of the line and the sheet of paper represents a portion of the plane.

Chart 1 - Activity 1 of the Activity Book 1

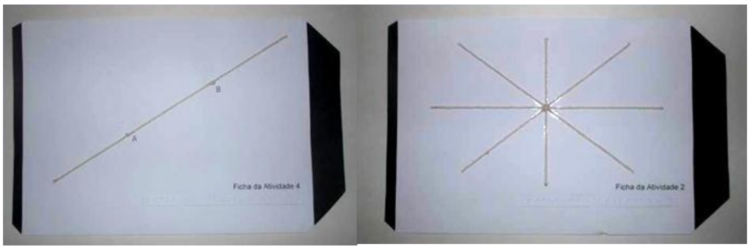
Presentation of Activity 1: Did you know that Geometry is the part of Math that studies the shapes and patterns of regularity found in nature? We usually do not realize, but the world around us is full of examples of geometric shapes and patterns that repeat themselves in a regular sequence. Can you remember that Braille writing is composed of a set of points? Did you know that a thin, well-stretched thread suggests the representation of a part of a straight line? A sheet of Braille paper represents a part of what could be a plane? In which other situations of your daily life you could give examples of a point, line and plane? ACTIVITY 1 Item 1) Stick a sheet of paper under the angle brackets of the support board and mark a point with the pencil. Item 2) Withdraw the sheet from the board, turn it and notice the point that you have marked. Item 3) Now, fold the sheet tightly on the point marked, so the paper is folded exactly on the mark of the point. Item 4) Unfold the sheet of paper and fold it two more times in different places but on the point marked. Item 5) Observe carefully the marks you have done when folding the paper. Item 6) How many folds could you do that are different from the ones you have already made, using the same point? Notice that the marks done by you on the paper when you folded it represent straight lines. You probably have realized that in one point there are a lot of straight lines. Could you say how many? Did you know that in one point there are infinite straight lines?

Source: Activities Book 1 created by the Geometry Teaching Lab

The activity described next aims at the introduction of the *1st Euclidean Axiom*. The student should put another sheet of paper on the support board and mark two points in different places. Then, he/she withdraws the sheet and folds it on the points marked, in a way that the fold is well-defined and passes on the two points marked. After this, the student is asked to unfold the paper and repeat the procedure. In this way, the student is led to notice that the fold is part of a straight line which continues beyond the limits of the paper and also that in these two distinct points there is only one straight line. This statement corresponds to the *1st Euclidean Axiom*.

Departing from the previous activity, now in the third experiment, we present to the student a chart with a representation, in high relief, of a straight line with two points and (Figure 4, picture on the left) with it, we will introduce the layout and the concepts of *straight line segment, ray, half-plane and angles*.

Figure 4 - Charts with representation in high relief of a line on two points and pencil of concurrent lines

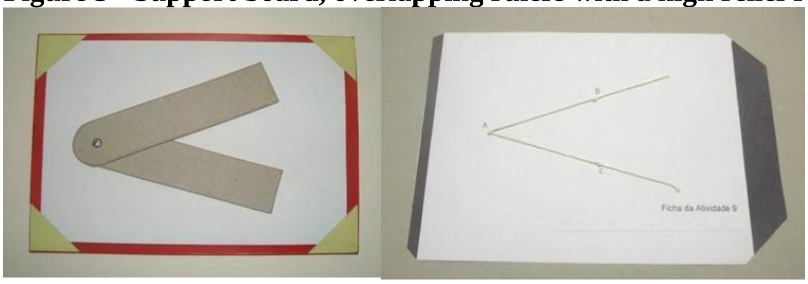


Source: Geometry Teaching Lab collection

With a common piece of thread we ask the student to tie some knots and to put the thread well-stretched on the board. After this, we ask the student how many knots can be tied in this thread. Therefore, the student is guided to associate this thread with a straight line and the knots with the points, and he/she can notice that there will be as many knots as he/she wants if there is more thread, concluding that a straight line has infinite points. This statement corresponds to the *2nd Euclidean Axiom*.

To introduce the concept of angle, in the fourth experiment, we use an artifact we call overlapping rulers (Figure 5, picture on the left). It allows movements and the rulers can be opened in any position on the plane of a table. We ask the student to put a sheet of paper on the support board and fix the ruler open in any position on it. Then, the student should draw a line in the inner border of both rulers. Finally, the student is supposed to withdraw the paper from the board and turn it, noticing the drawing in highrelief and then, to compare this drawing with a representation of a cardboard chart presented go him/her. (Figure 4, picture on the right). With this procedure, we expect that the student understands the concept of *angle*.

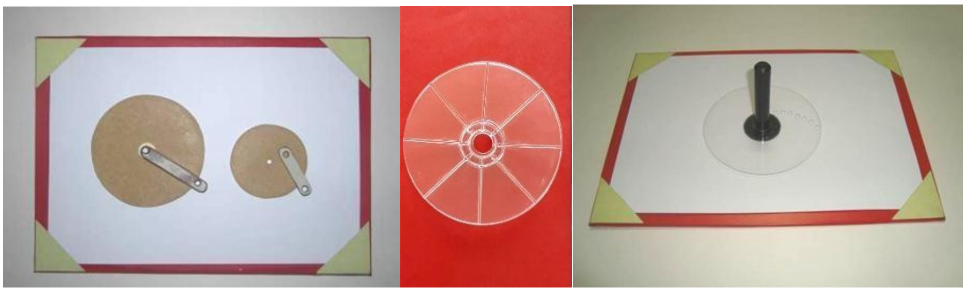
Figure 5 - Support board, overlapping rulers with a high relief representation



Source: Geometry Teaching Lab collection

The second Activities Book begins with the fifth experiment, with a rigid plastic disc and a chart with a high relief representation of a circumference; we present the concept of *circle and circumference* and their elements (Figure 6). In the sixth experiment, the student works with a set of artifacts which allow him/her to draw circumferences. It is formed by two round wooden pieces of different sizes, with a hole in the middle; one piece of plastic material and two wooden strips with two holes, one in each extremity (Figure 6, on the left).

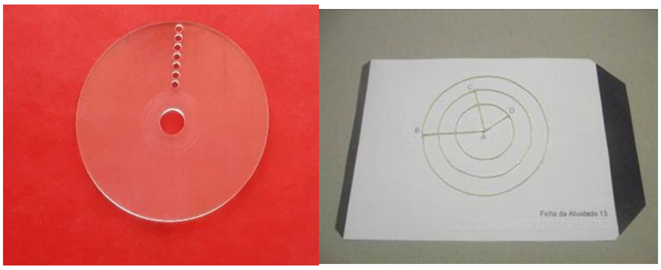
Figure 6 - Wooden disc with holes in the middle on the support board; rigid plastic disc with its respective radiuses and rigid plastic disc with a plastic support peg



Source: Geometry Teaching Lab collection

In this experiment we ask the student to fix one of the wooden pieces on a sheet of paper, which will be on the support board, without concealing the central hole. We advise the student to put the tip of the pencil in this hole, marking a point. Keeping the wooden piece fixed on the paper, we ask the student to fit the plastic piece in the central hole. Finally, with the tip of the pencil in the remaining hole of the wooden strip, we ask the student to draw a line contouring the round wooden piece. The student must then withdraw the sheet of paper from the board and, turning it, will be able to notice the drawing he/she has made in high relief, comparing it with the cardboard chart presented (Figure 7, on the right). Then, we ask the student to follow the same procedure with the other disc on another sheet of paper. After this, he/she must put both sheets side by side, and imagine they form a unique portion of the plane. Therefore, the student can realize, using the sense of tact, that the circumferences do not have the same center or the same radius. Doing this we can establish that: with any given point in the plane and with any radius it is always possible to draw a circumference on the plane. This statement corresponds to the *3rd Euclidean Axiom*.

Figure 7 - Rigid plastic disc with holes and chart with a high relief representation of concentric circumferences



Source: Geometry Teaching Lab collection

The set of artifacts of the 7th experiment allows us to draw the representation of concentric circumferences (Figure 7). In this phase, we lead the student to notice that the plastic disc has several small holes and a central, bigger one. We ask him/her to make a new object in the following way: first, the student needs to put the central hole of the disc in the plastic support peg and then, the syringe. After this, the student has to fix this object on the paper of the support board and putting the tip of the pencil in the small hole, he/she must rotate the disc around the syringe (Figure 6, on the right). We ask the student to repeat the procedure using all the other holes. After doing that, the student has to withdraw the sheet of paper from the support board and turn it to the other side in order to notice the high relief drawing he/she has just made. Besides, the student can compare it with the cardboard chart presented. By the end of this task, the student will be able to notice that these circumferences have the same center, but their radiuses have different sizes. With these procedures, he/she may understand the concept of *concentric circumferences*.

The 3rd Activities Book presents the 9th experiment, bringing to the student the concept of perpendicular lines and pencil of parallel lines. The student has to handle with the cardboard strips and paper clips (Figure 8 on the right). We ask the student to put the smaller strip on the smaller side of the board, fixing it with the paper clips. The student is asked to put a ruler on this strip and draw the representation of a straight line. After this, he/she must do the same procedure putting the bigger cardboard strip on the bigger side of the board, drawing the representation of a straight line. Then the student removes the sheet of paper from the support board and, turning it, may notice the high relief drawing he/she has just made. The student must also compare his/ her drawing touching the one on a cardboard chart put in front of him/her (Figure 8, central picture). Therefore, we lead the student to realize that the representations that he/she has drawn are *perpendicular lines* and the angles formed are *right angles*. Besides, the *4th Euclidean Axiom* is introduced, with the statement that two right angles are always equal.

Figure 8 - Support board with cardboard strips and paperclips, chart with a high relief representation of perpendicular lines and chart with two "dolls"

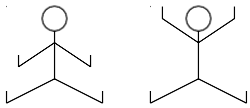
Source: Geometry Teaching Lab collection

In order to introduce the concept of parallel lines, we ask the student to fix the smaller cardboard strip on the smaller side of the support board, as in the previous experiment. We ask him/her to put a ruler onto the strip and draw the representation of a straight line. After this, the student must slide the ruler, keeping it onto the strip, and draw other representations of straight lines. After this, he/she must withdraw the sheet of paper and, turning it, may realize the drawing in high relief made, and compare it with the shape drawn on the cardboard chart presented. In this procedure we lead the student to notice the representation of a pencil of parallel lines created in high relief.

Besides, he is questioned about how many parallel lines pass in the same point out of a determined straight line. The student may realize that there is only one possible parallel line in this point. Therefore, the student will be introduced to the statement of the 5th Euclidean Geometry Postulate or the Straight Lines Postulate, by the mathematician and physician John Playfair (1748-1819): "In a point of the plane not belonging to a determined straight line, there is one, and only one parallel line considering the determined straight line."

Finally, in the last experiment, and aiming at the introduction of the important concept of direction, which several times is not known even by teachers, we present to the student a cardboard chart with a high relief representation of two "dolls" and we invite them to analyze the position of "feet" and "hands" (figure 9).

Figure 9 - "Dolls"



(a) *(b)*

Figure 9. "Dolls"

6. Some reflections about applying these experiments

We would like to report how these experiments have been taken by the visually impaired students, as they were applied to a student with low vision and six blind students, aged between 15 and 22, in the 1st and 2nd years of High School. In order to do so, there were two sessions of two hours each, guided by three members of the Geometry Teaching Lab team.

The students approved of the support board because it offers, in its corners, angle brackets to fix the Braille paper. The said that some drawing boards they had already had contact with, did not offer resources to fix the paper. Referring to the adequacy for the use of this resource, a blind student (16 years old) said: *"I loved the board!".*

The cardboard charts with drawings in high relief were well-accepted by the blind students, but one students with low vision (aged 22) observed that the contrast of colors is inadequate. He pointed out that the drawing in high relief and the paper had almost the same tone of color and he could not see it. He suggested that we changed the color of the drawing so it would contrast the color of the paper. Doing so, the students with low vision would be able to see the drawings.

Referring to the activities of drawing various circumferences, a blind student (aged 16) said: *"I liked the idea, I thought it was really creative, nice, and it can help a lot. [...] it gives a good notion of when we can move things, so it is easier for our comprehension".*

Another blind student (also aged 16) commented: *"I liked the wooden discs very much, despite not being able to do as many circumferences as we do with the plastic disc. But we can have an idea. One reinforces the other. We are better able to use these discs than the compass, because there is something fixed. We can leave the pencil a little bit and hold only the plastic peg or leave the peg and hold the pencil, differently from the compass".*

Also, the same student with low vision (aged 22) commented: *"I loved the wooden discs, it's cool to do the circumferences".*

Figure 10 - Blind students drawing circumferences with discs on the support board at the museum; teaching resources in exhibition at the LEGI museum

Pictures: Geometry Teaching Lab collection

After participating of the activities about the Euclidean Axioms, a blind student (aged 15) emphasized that she finally understood, during the experiments, what were perpendicular lines. It happened when she drew perpendicular and parallel lines using the cardboard strips, and she said: *"That is cool! Are these perpendicular lines?"* We noticed that the enthusiasm she demonstrated when touching the high relief drawing, was strong because she had never been exposed to a tactile model of these straight lines. She told us that her teachers only described the straight lines orally and she imagined them. Apparently that was the first time that the student encountered a figure with a cross shape, as the one presented in high relief. This fact is shocking, when we think that the majority of the Brazilian population professes religions which are based on Christianism, whose major symbol is a cross.

7. Final Considerations

We can state that the diversity of the manual teaching materials created by the Geometry Teaching Lab allow the teachers, even with little financial resources, to transform their classrooms and make them a ludic environment, motivating and inclusive. This can contribute to a better acceptance of Math in schools, and the discipline may reach a higher number of students. These resources may lead the students who can see or are visually impaired "not only to see Math in school with their hands, eyes and mind, but also see it with their heart" (Kaleff A. M., 2016b, p. 210).

It is also important to point out that we understand inclusion as a movement that is not restricted to disabled people, it is extended to everyone: "It is necessary to deconstruct the patterns and social conventions imposed by those who consider themselves the majority. It is also fundamental that we break the prejudice and the social construction of deficiency […]" (Rosa, 2017, p. 25). There is no possibility of thinking about homogenous classrooms in which one teaching approach will solve all the difficulties:

> "There are differences and similarities – not everything in schools must be equal, as well as not everything must be different [---] it is necessary that we have the right to be different when equality distorts the human beings and the right to be equals when the difference undermines us. (Araujo, Arantes, Klein, & Pereira, 2007, p. 16)

There are difficulties, but we cannot consider only the legislation, we need more actions in order to see that the real meaning of the word inclusion prevails in schools and in Math education. Schools need to overcome the idea of homogeneity and search, with all commitment and equity, for an inclusion that really means respect to the equality of rights of everyone. In other words, all people should have access to education according to their needs. Everyone must be included, independently from their abilities and difficulties, with respect to their singularities, aiming at a meaningful learning process.

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