Mathematics In The Dark

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In 1749 Diderot published his *"Letter on the blind for the use of those who see"* in which he discussed on some of the most interesting philosophical debates of the eighteenth century about the perceptual origin of abstract ideas (Diderot, 1749). Diderot had observed blind people, and deaf people, and people with speech impairments, to think about sensations.

I like to understand Diderot's letter to the blind as a tribute to human diversity, and as a beautiful historical source which demonstrates how our own knowledge is dependent on the experience of others who live and see things differently. I write this talk with the aim of sharing some of the recent, unfinished research to which I devoted the last years of my academic activity. It has to do with re-thinking our knowledge on mathematics teaching from the perspective of the learning abilities of those having difficulties, and among them blind students, who learn in the dark. From my experience with blind students and teachers I have learnt how a deficit, in this case the absence of vision, emphasizes and informs other fundamental, general aspects of mathematics teaching. Therefore, my focus is not on thinking how to teach mathematics to students with special needs, nor on finding ways to support them, but on how to use their way of learning to rethink some aspects, at least, of our knowledge on teaching.

First, I will discuss with you some excerpts of the experience of a mathematics classroom with a blind teacher and three students, two of whom are also totally blind. Unfolding this teaching experience will permit me to argue that learning difficulties or special needs afford rich opportunities for learning which are not always present elsewhere. The classroom episodes that we are going to discuss here are analyzed in detail in Figueiras & Arcavi (2014, 2015). The following are the main conclusions arising from that analysis:

- Blind students develop specific strategies to connect global and local properties, as well as topological properties concerning the inside and outside of objects. Local properties are observed in the neighborhood of a point, and usually blind students explore them with the fingertips. Tactile exploration strategies and their role in reasoning and learning mathematics are seldom found in the mathematics education literature or in the design of mathematical activities for the classroom.
- Blind people face several difficulties in creating and following chains of symbolic

developments, like solving an equation. Thus the blind student may tend to develop alternative ways to avoid these difficulties, which are afforded explicitly by the teacher. Students are guided to use logical reasoning that compels them to invoke images generated beforehand and connect concepts and definitions, instead of going right away to a symbolic calculation for that purpose.

- When observing blind teachers and students one may realize the central role that language and references to everyday objects and situations, even purely visual, play in doing mathematics. The combination of visual and non-visual metaphors, and of these again with haptic experiences, seems to act as an essential component in the representation of mathematical ideas.
- Contrasting how the blind teacher addresses the student with residual eyesight and the totally blind student, we realize that, in the presence of vision, it is assumed that the visual image speaks mathematically for itself. When talking to the first student, explanations are briefer, leaving some mathematical elements implicit. In contrast, when talking to the second one, explanations include richer verbal descriptions and haptic experiences related to those elements. Moreover, explanations to the first seem to be more impersonal and to have a poor argumentative structure, while in the second case a perlocutionary effect is clearly at play to convince and engage the listener in thinking and visualizing.
- Related to the last, the lack of vision seems to promote attentive listening as yet another resource for knowledge construction.

Summing up, knowledge construction and mathematical reasoning of blind people is supported by the combination of other resources, such as haptic perception, limited use of symbolic algebra, powerful verbal descriptions and attentive listening which could easily be incorporated as resources for general sighted settings.

I have used these examples to show that observing how visually impaired people deal with mathematical tasks is a fruitful source of insights on teaching and learning. This includes valuing the powerful interdependence of visual and haptic perception; knowing about the many possibilities that haptic experiences afford in mathematics teaching, or offering a different perspective on the role of language and algebra in supporting mathematics learning.

However, if we accept the thesis that knowing about the experiences of those with special needs helps us to re-think and conceptualize some aspects of teaching differently, special education and diversity also impose constraints that do not make it always possible.

In the following, I will discuss this issue building up from other teaching experiences which contrast to the previous ones. We have taken from the class with the blind students positive resources to enlarge our knowledge on teaching-related issues, while in the example coming up next, positive resources coming from students' interactions are eclipsed. I will try to support the thesis that these resources are not visible probably because the class is planned as a class for students having learning difficulties, and learning difficulties are associated with low expectations of students' potential for achievement.

I will show you two excerpts of a mathematics class with the same teacher teaching the two groups of students into which her whole class has been split. The decision of splitting the group was a school decision and the division had been made at the beginning of the year, separating from the whole class a small group of students having difficulties, identified on the basis of previous-year reports of the students' achievement.

The videotaped classes take place on consecutive days. The topic that is being studied is proportionality, and both classes start by analyzing a problem in the same context, a Formula 1 race in which Hamilton and Alonso run. The aim is to find a relationship between the number of laps of a F1 circuit and the amount of fuel used. The planning of the session, the initial description of the content, and the statement of the problem that is going to be solved are the same because the teacher is very committed to teaching both groups the same content. However, we are surprised by a very different qualitative unfolding of the two classes:

- Norms are different: in one of the classes, the students are allowed to interrupt freely to ask questions anytime, while in the other class they can ask only at specific moments after listening to the teacher's explanations.
- Opportunities for collaborative work are different: in one of the classes the teacher affords opportunities to discuss and work in groups, while there is no evidence of this offer in the other class.
- The expression of value given to student ideas is different: When responding to the same kind of student contribution, the teacher values it as "fantastic" or "extraordinary" in one class, while it is valued as "good" or "correct" in the other one.
- The public or private management of the same kind of student contribution is different: In one class, students' observations are considered interesting and openly

discussed as a learning opportunity for the whole class, while in the other class they are addressed in private conversation with the student who asked.

- Connections among mathematical concepts are different: In one class, the objective is to use ratio for describing linear change, while in the other, the aim is to find the particular ratio for solving the particular problem.

One plausible interpretation that makes sense to me from the perspective of other cases analyzed by other researchers with whom I have collaborated has to do with teachers' expectations of students' skills (Straehler, Pohl, Fernández, Gellert & Figueiras, 2013). Expectations operate implicitly and result in different conditions and learning opportunities, particularly with a different image of mathematics, for instance, through the creative and more powerful use of a general linear modeling in the first class, or as a static, instrumental view of it and a set of techniques to be applied in the second class.

Many times these expectations are self-fulfilling, in the sense that they can be defined as an anticipation of how a situation will develop, which then prompts those who are involved to behave so that these expectations occur (Merton, 1948). They take place in a very implicit, deep way, and build a strong obstacle for attention and inclusion of students with special needs because they trigger a high qualitative difference among mathematical practices. Having low expectations of students' achievement leads in general to offering limited opportunities to engage in mathematical practices even when this is consciously not the intention. One of the best ways to improve our teaching is taking those expectations seriously, devoting time to reflect on their origin, how they implicitly grow, and how they affect our practices. Low expectations prevent us from actively considering the many times distinctive potential resources that students with difficulties might bring. Reflecting on these expectations from the point of view of the learning opportunities that special needs offer is in my view a good exercise for mathematics teachers to improve their own professional knowledge. An improvement in the quality of the mathematics education of students with special needs as a consequence of such reflection is, at least, plausible.

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